MATH 3A MIDTERM 1 REVIEW

You should also do the practice problems on webwork. You should ask if you have questions about these problems. Solutions will NOT be posted.

I. Limit

1. Compute the following limits.

a) $\lim_{y\to 10} \frac{y^2-y+1}{y-5} =$ b) $\lim_{t\to 3} \frac{t^2-9}{t^3-27} =$ c) $\lim_{x\to a} \frac{a-x}{\sqrt{a}-\sqrt{x}} =$ ($a > 0$ constant)

2. Suppose $f(x)$ is a function continuous on the interval $[0, 10]$ and

$$\lim_{x\to 1} \frac{f(x)+1}{2} = 1.$$

What is $f(1)$?

II. Computing derivatives

A. Limit definition

$$f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h} \text{ or } \lim_{x\to a} \frac{f(x)-f(a)}{x-a}.$$

3. The following limits represent the derivative for some function $f(x)$ at the point $x = a$. Identify the function $f(x)$ and the value of a.

a) $f'(a) = \lim_{h\to 0} \frac{(4+h)^3-64}{h}$

b) $f'(a) = \lim_{x\to 1} \frac{\ln x}{x-1}$

4. Use the limit definition of derivative to compute $f'(x)$ for $f(x) = \frac{1}{x+2}$.

B. Differentiation rules

Power rule	Exp/Log functions	Product rule	Chain rule	Trig functions
$(x^n)' = nx^{n-1}$	$(e^x)' = e^x$	$(fg)' = f'g + fg'$	$(f(g(x)))' = f'(g(x)) \cdot g'(x)$	$(\sin x)' = \cos x$
	$(a^x)' = (\ln a)a^x$	Quotient rule	or $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$	$(\cos x)' = -\sin x$
	$(\ln x)' = \frac{1}{x}$	$(\frac{f}{g})' = \frac{f'g-fg'}{g^2}$		$(\tan x)' = \sec^2 x$

5. Compute the derivatives of the following functions at the given point.

a) $f(x) = (3x^2 - 2x + 10)e^{2x}$, $x = 1$

b) $f(x) = \frac{-4x^2+x}{(1-x)^2}$, $x = 0$

c) $f(x) = \sqrt{\sin(x^3) + 4 - e^{x^2}}$, $x = 0$

d) $f(x) = \cos(x^2 e^{\pi})$, $x = \pi$

6. Use the rules given above to derive that $(\sec x)' = \sec x \tan x$.

III. Tangent line

Let $(a, f(a))$ be a point on $y = f(x)$. The tangent line to $f(x)$ at $x = a$ is given by the equation

$$y = f'(a)(x-a) + f(a).$$

This is simply the point-slope form of equation of a line with $m = f'(a)$.

7. Find the equation of tangent to the curve $f(x) = \frac{-2}{(3x+1)^2}$ at $x = 0$.

8. Suppose $f(x)$ is a differentiable function and

$$\lim_{x\to 3} \frac{f(x)+10}{x-3} = 2.$$

What is the equation of tangent to $f(x)$ at $x = 3$?

IV. Linear approximation

Linear approximation means you use the equation of tangent to approximate the function. Basically, this means you plug x into the equation of tangent instead of the original function to find y.

9. a) Find the equation of tangent to $f(x) = \ln x$ at $x = 1$.
b) Use part a) to approximate $\ln(1.1)$.
10. Use a linear approximation to estimate $\cos(\frac{3}{4})$.
(What would you use for $f(x)$? What should you pick to be the base point a?)

V. Implicit differentiation

You use implicit differentiation when you can't solve y in terms of x explicity. You want to keep in mind that $y = f(x)$ is a function of x and use chain rule.

11. Find dy/dx if
a) $y^2 = xy + x^2 + x$
b) $\sin(xy) = e^y + \ln(x)$
12. Find the slope of tangent line to the curve

$$\frac{y-1}{x^2-y} = e^{2x} + 1$$

at the point $x = 0$. (Hint: What is y at this point?)

VI. Related rates

This is an application of (implicit) differentiation. The general steps to approach a related rate problem is
1) write down an equation relating the variables;
2) differentiate the equation;
3) plug in known values and solve for the unknown.
The variable you want to differentiate w.r.t. depends on what you are finding (e.g. if you need dy/dt then you would differentiate w.r.t. t so you have dt at the denominator).

13. The length of a rectangle is increasing at a rate of 8cm/s and its width is decreasing at a rate of 3cm/s. When the length is 20cm and the width of 10cm, how fast is the area of the rectangle changing? Is it increasing or decreasing?
14. At noon, ship A is 150km west of ship B. Ship A is sailing east at 35km/h and ship B is sailing north at 25km/h. How fast is the distance between the ships changing at 4pm?
15. If a snowball melts so that its surface area decreases at a rate of 1cm^2/min, find the rate at which the diameter decreases when the diameter is 10cm.
