MATH 3A MIDTERM 2 REVIEW

PP = Practice Problem

I. Graphing

Major concepts: 1st derivative - increasing/decreasing, 2nd derivative - concavity, locations of local extrema

1. Let $f(x) = 2x^3 - 3x^2 - 12x$.

a) Find where f is increasing/decreasing.
b) Find all local maxima/minima.
c) Find where f is concave up/concave down.
d) Find all inflection points.
e) Sketch the graph of f.

2. Sketch the graph of a function that satisfies all of the following conditions.

i. $f'(0) = f'(2) = f'(4) = 0$
ii. $f'(x) > 0$ if $x < 0$ or $2 < x < 4$
iii. $f'(x) < 0$ if $0 < x < 2$ or $x > 4$
iv. $f''(x) > 0$ if $1 < x < 3$
v. $f''(x) < 0$ if $x < 1$ or $x > 3$

II. Max/Min

Major concepts: locations of local extrema, 1st derivative test, locations of global extrema

3. Find all local maxima and local minima of $f(x) = x\sqrt{x+3}$ $(x > -3)$.

4. Find the absolute maximum and minimum of $f(x) = 2 + 2x^2 - x^4$ on the interval $[0, 2]$.

(PP20) 5. Find the point on the line $-5x + 7y - 1 = 0$ which is closest to the point $(-1, 0)$.

(PP22) 6. A Norman window has the shape of a semicircle atop a rectangle so that the diameter of the semicircle is equal to the width of the rectangle. What is the area of the largest possible Norman window with a perimeter of 26 feet?

7. A box is to be made with an open top, vertical sides, a square bottom, and a volume of 4m^3. The bottom costs $\$10/\text{m}^2$ and the sides cost $\$15/\text{m}^2$. Find the dimension of the most economical box. How much does it cost?

(PP30) 8. The top and bottom margins of a poster are 8cm and the side margins are each 6cm. If the area of the printed material on the poster is fixed at 388cm^2, find the dimensions of the poster with the smallest area.

III. Related Rates

(PP19) 9. A spherical balloon is inflated so that its volume is increasing at the rate of $2.4\text{ft}^3/\text{min}$. How rapidly is the diameter of the balloon increasing when the diameter is 1.2 feet?

(PP10) 10. A rock is thrown into a still pond and causes a circular ripple. If the radius of the ripple is increasing at a rate of 2ft/sec, how fast is the circumference changing when the radius is 18ft?

(PP16) 11. The radius of a right circular cone is increasing at a rate of 5in/sec and its height is decreasing at a rate of 4in/sec. At what rate is the volume of the cone changing when the radius is 50in and the height is 30in?

IV. (Slightly More) Challenging Problems

12. A variation of a triathlon competition has a contestant swimming from a point, A, on one shore of a lake, to a point C on the opposite parallel shore, then running to the finish, B, further along the lakeshore. The lake is 4km wide and the finish line is 10km down the lake. If a contestant can swim at 2km/hr and run at 10km/hr, determine the point C that will minimize the total time for the race.

13. The base of a pyramid-shaped tank is square with sides that are 9m in length. The vertex of the pyramid is 12m above the base. The tank is filled to a depth of 4m and water is flowing into the tank at a rate of $3\text{m}^3/\text{sec}$. Find the rate of change of the depth of the water at this moment. (Hint: Volume of a pyramid $=$ base area $\times$ height $/3$).
