Math 3A Differentiation Rules and Linear Approximation

Notations.

I. Differentiation Rules

0. "Sum rule" and "Multiplication by a constant rule"

1. "Constant rule"

2. "Linear function rule"

3. Power rule

Example. $f(x) = 3x^3 - \frac{1}{x} + \sqrt{x} + 1$

4. Exponent rule

Example. $f(x) = e^x + 2^x + x - 10$

5. Product rule

Example. $f(x) = (2x^4 - 3x^2)(2\sqrt{x} + \frac{2}{x^2})$

6. Quotient rule

Example. $f(x) = \frac{x^2+e^x+1}{2\sqrt{x}}$

7. Chain rule

Example. $f(x) = (2x^2 + 1)^{20}$

Example. $f(x) = e^{(4-3x^2)^2}$

II. Tangent line approximation

Example. Estimate $\sqrt{4.1}$ using a linear approximation of an appropriate function.

III. Practice

1. Find the derivative of the following functions. You do not have to simplify your answer.

a) $y = -2x^4 + x^2 + 4x\sqrt{x} - 10$
b) $y = x^2e^x + 1$
c) $y = \frac{5}{(x^2+4x-1)^9}$
d) $y = \frac{x}{\sqrt{4x+x^2}}$
e) $y = e^{e^{x}} - x$

2. Consider $f(x) = 3x^2 + 2x - 1$. The line $y = 8x - 4$ is tangent to the graph of $f(x)$ at the point $x = a$. What is the value of a?

3. a) Find the tangent line to the function $y = e^{2x}$ at $x = 0$.
b) Use part a) to estimate the value of $e^{0.2}$.

4. Use chain rule and the fact that $(e^x)' = e^x$ to show that $(a^x)' = (\ln a)a^x$ for $a > 0$.
(Hint: You can write $a^x = e^{\text{somthing}}$.)

5. A challenging problem:

You MUST use the limit definition of derivative to do this problem because the functions here are not defined by a single formula near $x = 0$.

a) Let $f(x)$ be defined by

$$f(x) = \begin{cases} x^2 \sin(\frac{1}{x}) & \text{for } x \neq 0 \\ 0 & \text{for } x = 0. \end{cases}$$

Does $f'(0)$ exist? If yes, what is it?

b) This time let $f(x)$ be defined by

$$f(x) = \begin{cases} x \sin(\frac{1}{x}) & \text{for } x \neq 0 \\ 0 & \text{for } x = 0. \end{cases}$$

Does $f'(0)$ exist? If yes, what is it?