**Math 3A Applications of Derivatives**

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## I. First Derivative

1. Increasing/Decreasing

2. Local Maximum/Local Minimum

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**Example.** Let $f(x) = \frac{x^3}{3} + x^2 - 3x + 1$.

a) Find the intervals on which $f$ is increasing and decreasing.

b) Find all local maxima and local minima of $f$.

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3. Absolute Maximum/Absolute Minimum

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**Example.** Let $f(x) = \ln(x^2 + x + 1)$. Find the absolute maximum and absolute minimum of $f$ on the closed interval $[-1, 1]$.

**II. Second Derivative**

Concavity

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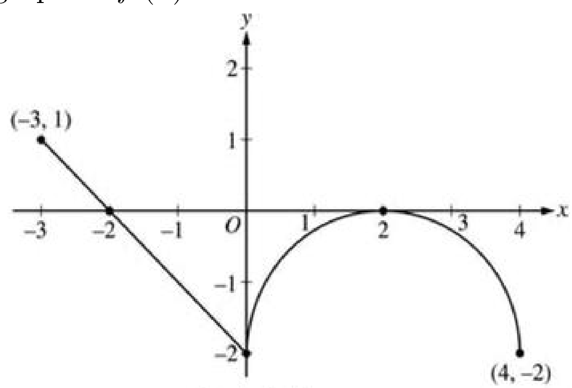
**Practice.** Below is the graph of $f'(x)$.



Graph of $f'$

a) When is $f(x)$ incresaing? When is it decreasing?

b) When does $f(x)$ have a local maximum? When does it have a local minimum?

c) When is $f(x)$ concave up? When is it concave down?

d) When does $f(x)$ have an inflection point?

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**Practice (Optimization).** Suppose you are taking a test with a total of 10 problems and you are told that your final score will be calculated as follows:

1. If you get $x$ problems right, then each correct problem will be worth $\frac{1}{10}(100 - x^2)$ points. (So the more you get right the fewer points each correct answer is worth.)
2. You get 2 bonus points for every problem you leave blank.

Suppose you know the materials really well that if you answer a problem you will get it 100% correct for sure. Also assume that $x$ can be in decimals since you can answer parts of a problem but not all.

a) How many problems should you answer to get the maximum number of points?
b) If the test is out of 60, what is the maximum percentage you can get on the test?

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**Practice (Optimization).** A cylindrical can is to hold $20\text{m}^3$ The material for the top and bottom costs $\$10/\text{m}^2$ and material for the side costs $\$8/\text{m}^2$ Find the radius $r$ and height $h$ of the most economical can.

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