MATH 3A L'HOPITAL'S, GROWTH/DECAYM MEAN VALUE THEOREM

I. L'Hopital's Rule

When does it apply? What does it say?

II. Growth & Decay

A. Model for Natural Growth/Decay

What is the model? What is the solution?

B. Model for Cooling (Newton's Law)

What is the model? What is the solution?

III. Mean Value Theorem

What is the statement of the theorem?

IV. Practice

1. Find
$$\lim_{x \to 1} \frac{\ln x}{\sin(\pi x)} =$$

2. Find
$$\lim_{x \to 0} \frac{\cos(ax) - \cos(bx)}{x^2} =$$

3. Find
$$\lim_{x \to 0} \frac{x}{e^{2x}} =$$

4. A bacteria culture initially contains 100 cells and grows at a rate proportional to its size. After an hour the population has increased to 420.

a) Find an expression for the number of bacteria after t hours.

b) Find the number of bacteria after 3 hours.

c) Find the rate of growth after 3 hours.

d) When will the population reach $10,000$?

5. A roast turkey is taken from an oven when its temperature has reached $185°F$ and is placed on a table in a room where the temperature is $75°F$.

a) If the temperature of the turkey is $150°F$ after half an hour, what is the temperature after 45 minutes?

b) When will the turkey have cooled to $100°F$?

Some cool problems you can solve using Calculus.

6. Show that the equation $2x - 1 - \sin x = 0$ has exactly one root.

7. Two runners start a race at the same time and finish in a tie. Assume that their position functions are smooth. Prove that at some time during the race they have the same speed.

*8. Show that the equation $x^4 + 4x + c = 0$ has at most two real roots.