MATH 3B FINAL REVIEW

Office Hours (SH 6431V): 3/15 Fri 5:15-7, 3/18 Mon 9-12 & 1-3:30, 3/19 Tue 9-12 & 1-4

1) For topics covered before arclength, see midterm reviews 1 & 2 and the two actual midterms.
2) The problems on this review are all very basic. You should know how to do all of them.
3) I won't post solutions to the practice problems; but you can email me if you need hints in solving them.

Useful link: http://math.ucsb.edu/oml/3B.html

I. Arclength

In terms of x : the length of the curve $y = f(x)$ on $a \leq x \leq b$ is

$$L = \int_a^b \sqrt{1 + [f'(x)]^2}dx.$$

In terms of y : the length of the curve $x = g(y)$ on $c \leq y \leq d$ is

$$L = \int_c^d \sqrt{1 + [g'(y)]^2}dy.$$

You can use either formula depending on which integral is easier to set up and integrate.

Ex1. Find the length of the curve $y = \frac{x^5}{6} + \frac{1}{10x^3}$ for $1 \leq x \leq 2$.
Ex2. Set up an integral to find the length of the curve $x = y + y^3$ for $1 \leq y \leq 4$. Then use a midpoint Riemann sum with $n = 6$ to estimate the integral.

II. Surface Area of Revolution

The area of a surface of revolution is

$$SA = \int_a^b 2\pi r ds,$$

where r is the radius, and ds is the rate of change of the arclength, i.e.

$$ds = \sqrt{1 + (\frac{dy}{dx})^2}dx \text{ or } \sqrt{1 + (\frac{dx}{dy})^2}dy.$$

You can use either formula for ds depending whether you want to set up the integral in terms of x or y. Also, the radius r would be x if the function is rotated about the y-axis, and y if about the x-axis.

Ex3. Find the area of the surface obtained by rotating the curve $y = 1 - x^2$ for $0 \leq x \leq 1$ about the y-axis.
Ex4. Find the area of the surface obtained by rotating the curve $y = x^3$ for $0 \leq x \leq 2$ about the x-axis.

III. Work

From Physics, we know that

$$W = Fd \text{ (work = force} \times \text{distance).}$$

If both the force and distance, then the work can be computer directly using this formula. But if at least one of them is changing, then we will need an integral.

A. Spring

Hooke's Law:

The work required to stretch a spring from a length of x_i to x_f beyond its natural length is

Standard units:

Example. Suppose that 2J of work is needed to stretch a spring from its natural length of 30cm to a length of 42cm.

a) How much work is needed to stretch the spring from 35cm to 40cm?
b) How far beyond its natural length will a force of 30N keep the spring stretched?

Ex5. A force of 10lb is required to hold a spring stretched 4in beyond its natural length. How much work is done in stretching it from its natural length to 6in beyond its natural length?

Ex6. If 6J of work is needed to stretch a spring from 10cm to 12cm and 16J is needed to stretch it from 12cm to 14cm, what is the natural length of the spring?

B. Cable/Rope

Example. A cable that weighs 2lb/ft is used to lift 800lb of coal up a mine shaft 500ft deep. Find the work done.

Ex7. A 10ft chain weighs 25lb and hangs from a ceiling. Find the work done in lifting the lower end of the chain to the ceiling so that it's level with the upper end.

Ex8. An 8ft chain weighs 120 pounds. A large robot is holding one end of the chain 3 feet above the ground, so that 5 feet of the chain are on the ground. How much work must the robot do to lift this end of the chain from a height of 3 feet to a height of 10 feet?

IV. Center of Mass

Let R be the region bounded above by $y = f(x)$ and below by $y = g(x)$ on the interval $[a, b]$.

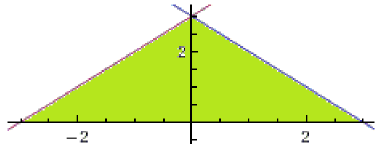
The moment of R about the y-axis is

The moment of R about the x-axis is

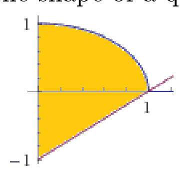
The coordinates of the centroid of R are given by

Theorem of Pappus:

Example (cf. HW#9 Problem 17) Calculus the moments M_x and M_y of the lamina below whose density $\rho = 5$.



Ex9. Find the centroid of the region bounded by $y = \sin x$ and the x-axis on the interval $0 \leq x \leq \pi$.
Ex10. Find the centroid of the lamina with the shape of a quarter-circle and triangle as shown below.



V. Average Value

The average value of a function $y = f(x)$ on the interval $[a, b]$ is defined to be

$$f_{ave} = \frac{1}{b-a} \int_a^b f(x)dx.$$

This is analogous to the formula

average = (sum of all the terms)/(total number of terms)

in the discrete case. Here the integral corresponds to "sum of all the terms" and $b - a$ to "total number of terms"

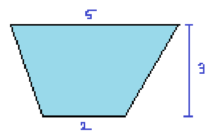
Ex11 . Find the average value of the function $f(x) = \ln x$ on the interval $[1, 3]$.
Ex12. Find the average value f_{ave} of the function $f(t) = t + \sin(4t)$ on the interval $[0, \pi]$. If $f(t)$ represents the velocity of a particle at time t, what is the physical meaning of the quantity f_{ave}?

VI. Hydrostatic Force and Pressure

The hydrostatic force and pressure on a plate of area A at a depth of d in a fluid with mass density ρ are given by

If one of these quantities is not constant, then we will need to use an integral.

Example (cf. HW#9 Problem 18). A verticale plate has the shape as shown below and is submerged in water. Assume that the measurements given are in ft. Using the fact that the weight of water is 62.5lb/ft^3, calculate the hydrostatic force (in lbs) against the plate.



Ex13. A tank is 8m long, 4m wide, 2m high, and contains kerosene with density 820kg/m^3 to a depth of 1.5m. Find a) the hydrostatic pressure on the bottom of the tank, b) the hydrostatic force on the bottom, and c) the hydrostatic force on one end of the tank.

Ex14. A large tank is designed with ends in the shape of the region between the curves $y = \frac{1}{2}x^2$ and $y = 12$, measured in feet. Find the hydrostatic force and pressure on one end of the tank if it is filled to a depth of 8ft with gasoline. (Assume that gasoline's density is 42lb/ft^3.)
