MATH 3B MIDTERM 1 REVIEW

Note: I will not post solutions or answers. Please email me or come to my office hours if you have questions. If you just want to check your answers, use wolframalpha.com.

I. Definite Integrals as Areas

Key: A definite integral can be interpreted as the area bounded by the grapg of the function and the x-axis. We count the area above the x-axis as positive, and the area below as negative.

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1. Evaluate the following integrals by interpreting it as an area.

a) $\int_{-3}^{5}(x+2)dx$
b) $\int_{4}^{10} 5dx$
c) $\int_{0}^{2}(1+\sqrt{4-x^2})dx$
d) $\int_{-1}^{8} |x-2|dx$
*e) $\int_{-4}^{4} \sin(x^3)dx$ (Hint: symmetry)

II. Riemann Sum

Key: A definite integral (area) can be approximated using rectangles where we use the function as our heights. As we use infinitely many rectangles (limit), we get the actual area.

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2. Estimate $\int_{1}^{4} \ln(2x+1)dx$ by using a left Riemann sum with $n = 3$. Is your approximation bigger of smaller than the actual value?

3. Estimate $\int_{1}^{7} \sqrt{x+4}dx$ by using a midpoint Riemann sum with $n = 4$. Is your approximation bigger of smaller than the actual value? (Hint: concavity)

4. Write down the Riemann sum definition for the integral $\int_{-1}^{4} \frac{\cos x}{x^2+e^x}dx$.

5. The Riemann sum

$$\lim_{n\to\infty} \sum_{i=1}^{n} \sin(\ln(1+\frac{i}{n}))\frac{1}{n}$$

represents the value of an integral $\int_{a}^{b} f(x)dx$. What are a, b, and $f(x)$?

III. Indefinite Integrals/Anti-derivatives

Key: An indefinite integral is derivative backwards, so we just need to think - what function do we need to differentiate to get the integrand?

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6. Find the following indefinite integrals by "thinking backwards".

a) $\int(x^2+2x-10)dx$
b) $\int(e^x+x^{-1})dx$
c) $\int(\sin x+\sec^2 x)dx$
d) $\int \cos(2x+1)dx$

*e) $\int xe^{x^2}dx$ (If you can't think backwards here, just use u-sub.)
*f) $\int \frac{3x^2}{\sqrt{x^3+1}}dx$ (If you can't think backwards here, just use u-sub.)

IV. Properties of Integrals

7. Find $\int_4^{10} f(x)dx$ if

$$\int_0^{10} f(x)dx = 5, \quad \int_4^{12} f(x)dx = -3, \text{ and } \int_{10}^{12} f(x)dx = -1.$$

8. Show that the following inequality holds or explain why this is false.

$$\int_0^1 \sqrt{1+x^2}dx \leq \int_0^1 \sqrt{1+x}dx.$$

9. Show that

$$2 \leq \int_{-1}^1 \sqrt{1+x^2} \leq 2\sqrt{2}.$$

V. Fundamental Theorem of Calculus Part 1

Key: Integration and differentiation are reverse operations.

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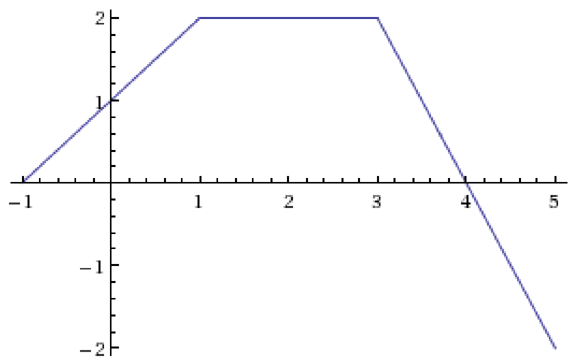
10. Let $g(x) = \int_1^x f(t)dt$ where $f(t)$ is a function whose graph is shown below.



a) Find $g(4)$, $g(5)$, and $g(-1)$.
b) On what interval(s) is $g(x)$ increasing?
c) Find all local extrema of $g(x)$.
d) On what interval(s) is $g(x)$ concave up?

11. Find the derivative of the following functions:

a) $f(x) = \int_{-1}^{x^2} e^u \ln(u^2)du$.
b) $g(x) = \int_{2x}^{0} \sin(t+1)dt$.
c) $h(x) = \int_{e^x}^{\sin(x^2)} \tan(\sqrt{t^2+1})dt$
*d) $r(x) = \int_{-1}^{5} e^{u+1} \ln(u^2)du$ (Hint: where is the variable?)

VI. Fundamental Theorem of Calculus Part2/Net Change Theorem

Key: A definite integral can be computed using the anti-derivatives. In particular, integrating a derivative over $[a, b]$ gives us the net change of the original function from $x = a$ to $x = b$.

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12. Evaluate the following definite integrals.

a) $\int_{-\pi}^{\pi/2} \cos(x)dx$.
b) $\int_1^2 \sqrt{x^3}dx$.
c) $\int_1^9 \frac{x-1}{\sqrt{x}}dx$.
d) $\int_0^{\pi/4} \sec\theta \tan\theta d\theta$.
e) $\int_0^{\pi} f(x)dx$ where $f(x) = \begin{cases} \sin x & \text{if } 0 \le x \le \pi/2 \\ \cos x & \text{if } \pi/2 < x \le \pi. \end{cases}$

13. Suppose a particle is moving along the x-axis whose velocity is given by $v(t) = x^2 - 3x + 2$.

a) Find the overall displacement of the particle from $t = 0$ to $t = 4$.
*b) Find the total distance traveled by the particle from $t = 0$ to $t = 4$.
c) What is the initial (i.e. when $t = 0$) velocity of the particle?
d) What is the initial (i.e. when $t = 0$) acceleration of the particle?
e) If the particle is at $x = 1$ initially, find a function which describes the position of the particle at any time t.

VII. U-substitution

Key: Think of u-substitution as chain rule backwards. Also, it transforms the integral into a new integral. So if you get really stuck, doing a u-sub might help.

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14. Find the following indefinite integrals.

a) $\int x\sin(x^2)dx$
b) $\int \cos\theta \sin^6\theta d\theta$
c) $\int \frac{2+\cos\sqrt{t}}{\sqrt{t}}dt$
d) $\int \frac{\tan^{-1} x}{1+x^2}dx$
e) $\int (2 + e^{\tan x})\sec^2 x dx$

15. Compute the following definite integrals.

a) $\int_1^2 \frac{e^{1/x}}{x^2}dx$
b) $\int_0^1 x^2(3x^3 - 2)^{20}dx$

VIII. Integration by Parts

Key: Usually, we pick u to be a function which gets simpler when we differentiate, and dv to be a function which we know how to integrate.

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16. Find the following indefinite integrals.

a) $\int x\ln(2x)dx$
b) $\int x^2\cos(4x)dx$
c) $\int e^{2x}\sin(3x)dx$
d) $\int x\sec^2(3 - x)dx$.

17. Compute the following definite integrals.

a) $\int_4^9 \frac{\ln y}{\sqrt{y}}dy$

b) $\int_{-1}^{4} x^2 e^{3-x} dx$
c) $\int_{1}^{2} x^4 (\ln x)^2 dx$

IX. Trickier Integrals

18. Find the following indefinite integrals by first doing a u-sub and then integration by parts.

a) $\int \sin(\sqrt{x}) dx$
b) $\int t^3 e^{-t^2} dx$
c) $\int \theta^3 \cos(\theta^2) d\theta$
d) $\int e^{\cos t} \sin(2t) dt$ (Hint: double angle formula for $\sin(2t)$)
e) $\int \sin(\ln x) dx$

19. Find the following indefinite integrals.

a) $\int \cos^2(x) dx$ (Hint: double angle formula for $\cos(2x)$)
b) $\int (\ln x)^2 dx$
c) $\int \frac{x \sin(x^2)}{1+(\cos(x^2))^2} dx$
d) $\int \frac{e^x}{e^{2x}+2e^x+1} dx$
e) $\int x e^{x^2} \sec^2(e^{x^2}) dx$
