MATH 3B MIDTERM 2 REVIEW

Office hours next week:

1) Mathlab hours (SH 1607): M 1-3 (as usual)
2) Office hours (SH 6431V): W 2:00-4:45 (extended)

I. Integrals of Trig Functions (Section 7.2)

A. Cosine and Sine

- both even: use double-angle formula
- one is odd: use u-sub
- angles are different: use product-to-sum formula

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1. Compute $\int \cos^2(3\theta)d\theta$.
2. Compute $\int \cos^3(4t)\sin^2(4t)dt$.
3. Compute $\int \cos^{19}(x)\sin^3(x)dx$.
4. Compute $\int \cos(10y)\sin(2y)dy$.

B. Tangent and Secant

- secant even: set $u = \tan(\cdot)$
- otherwise: set $u = \sec(\cdot)$
- might need to use the identity $1 + \tan^2\theta = \sec^2\theta$
- if u-sub doesn't work, need to try something else

5. Compute $\int \sec^4(2\theta)\tan(2\theta)d\theta$.
6. Compute $\int \sec^3(3x)\tan^3(3x)dx$.
7. Compute $\int \tan^3(t)dt$.

II. Trig Substitution (Section 7.3)

- $\sqrt{a^2 - x^2}$: set $x = a\sin\theta$ and use the identity $1 - \sin^2\theta = \cos^2\theta$.
- $\sqrt{x^2 + a^2}$: set $x = a\tan\theta$ and use the identity $\tan^2\theta + 1 = \sec^2\theta$.
- $\sqrt{x^2 - a^2}$: set $x = a\sec\theta$ and use the identity $\sec^2\theta - 1 = \tan^2\theta$.
- use a triangle at the end to convert your answer back in terms of x
- might need to do completing the square before substitution

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8. Compute $\int \frac{x^3}{\sqrt{16-x^2}}dx$.
9. Compute $\int \frac{u}{\sqrt{u^2-7}}du$.
10. Compute $\int \frac{dx}{(9x^2+4)^{3/2}}$.
11. Compute $\int \frac{y}{\sqrt{y^2+y+1}}dy$.

III. Partial Fractions (Section 7.4)

- use this technique when you have a product of polynomials at the denominator
- do long division when deg(numerator) $\geq$ deg(denominator)

- some useful identities:

$$\int \frac{dx}{x+a} = \ln|x+a| + C, \quad \int \frac{x}{x^2+a^2}dx = \frac{1}{2}\ln|x^2+a^2| + C, \quad \int \frac{dx}{x^2+a^2} = \frac{1}{a}\arctan(\frac{x}{a}) + C$$

This is true for any constant $a \neq 0$ and you should understand why these are true.

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12. Compute $\int \frac{x-9}{x^2+3x-10}dx$.
13. Compute $\int \frac{10}{(x-1)(x^2+9)}dx$.
14. Compute $\int \frac{x^2+2x-1}{x^3-x}dx$.
15. Compute $\int \frac{x^3+x}{x-1}dx$.

IV. Improper Integrals (Section 7.8)

- improper if $\pm\infty$ is in the bound or the function is undefined somewhere on the interval
- split the integral if necessary so that there is exactly one "bad point" in the bounds
- need every individual integral to converge for the original integral to converge
- integrate as usual and take the limit at the end

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16. Compute $\int_0^\infty xe^{-4x}$ or explain why it diverges.
17. Compute $\int_{-\infty}^\infty \frac{x}{1+x^2}dx$ or explain why it diverges.
18. Compute $\int_0^2 z^2 \ln z dz$ or explain why it diverges.
19. Compute $\int_{-1}^1 \frac{dx}{\sqrt{1-x^2}}$ or explain why it diverges.

V. Areas Between Curves (Section 6.1)

- use dx or dy depending on how you cut the region
- you might need to find the bounds yourself

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20. Find the area of the region bounded by the curves:

a) $y = 12 - x^2$ and $y = x^2 - 6$.
b) $y = \sqrt{x}$, $x = -1$, and $y = 4$.
c) $y = \sin x$ and $y = \cos x$ on the interval $[0, \pi/2]$ (hint: you will need two integrals).

VI. Volumes of Revolution (Sections 6.2 & 6.3)

- Washer: the volume of a typical washer is $\pi(R^2 - r^2)$(thickness)
- Shell: the volume of a typical shell is $2\pi rh$(thickness)
- you can find the R, r, h and thickness from the picture
- the thickness will be dx or dy depending on how you cut the region

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21. Find the volume of the solid obtained by rotating the region bounded by the curves:

a) $y = 1 - x^2, y = 0$; about the x-axis
b) $y^2 = x$, $x = 2y$; about the y-axis
c) $y = x$, $y = \sqrt{x}$; about $x = 2$
d) $y = \frac{1}{x}$, $y = 0$, $x = 1$, $x = 3$; about $y = -1$
