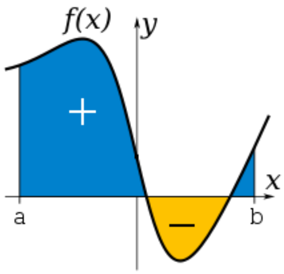
MATH 3B INTEGRAL AND RIEMANN SUM

I. Integrals

What does the symbol $\int_a^b f(x)dx$ represent?



Examples. Find the following integrals using the above interpretation.

$$\int_{-1}^{2} xdx = \qquad\qquad \int_{-2}^{2} \sqrt{4-x^2}dx =$$

II. Approximating an Integral

What about $\int_{-3}^{10}(x^2+10)dx$?

Idea: Estimate the area using regular shapes.

Riemann sum: Estimate the area using rectangles whose base lengths are all the same.

Example. Use a left Riemann sum with $n = 4$ to estimate $\int_1^3 \ln x dx$.

Is the estimate bigger or smaller than the actual area?

IV. Practice

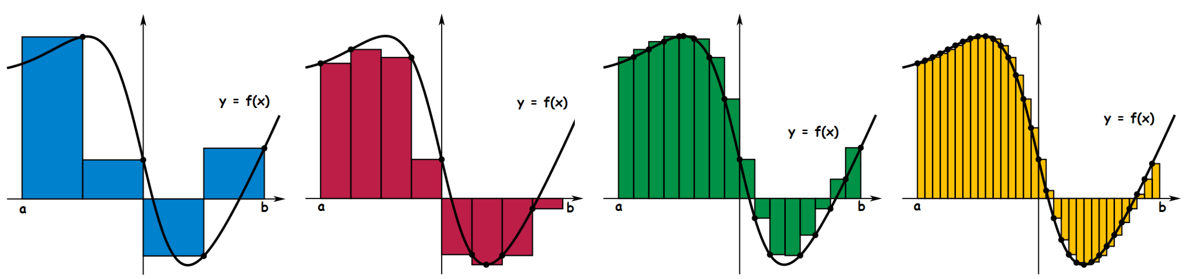
1. (Right Riemann Sum) Use a right Riemann sum with $n = 3$ to estimate $\int_{-1}^2 e^x dx$. Is the estimate bigger or smaller than the actual area?

2. (Upper Riemann Sum) Use a Riemann sum with $n = 3$ to estimate $\int_0^\pi \sin x dx$. For each subinterval, use a point which will give you an upper bound of the integral (i.e. the estimate will be bigger than the actual value). What if you want to get a lower bound?

3) (Riemann Sum with Two Functions) Use a midpoint Riemann sum with $n = 4$ to estimate $\int_0^1 (\sqrt{x} - x)dx$ (the area below $\sqrt{x}$ and above x on the interval $[0, 1]$)?

V. Riemann Sum (Actual Area)

Idea: The estimation gets better and better as we use more rectangles. So we can get the actual area using "infinitely many" (let $n \to \infty$) rectangles in the Riemann sum.



The formula is

$$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(a + i\Delta x)\Delta x, \text{ where } \Delta x = \frac{b-a}{n}.$$

How do you make sense of this formula? In terms of the picture, what do the following quantities/symbols represent? Can you label them in one of the pictures above?

1) n
2) Δx
3) i
4) $x + i\Delta x$
5) $f(x + i\Delta x)$
