# MATH 3B INTEGRALS AND FUNDAMENTAL THEOREM OF CALCULUS

## I. Definite Integrals

### A. Basic Properties

1. $\int_a^a f(x)dx =$
2. $\int_a^b f(x)dx =$
3. $c\int_a^b f(x)dx =$
4. $\int_a^b [f(x) + g(x)]dx =$
5. $\int_a^b [f(x) - g(x)]dx =$

### B. Other Properties

**Exercise:** Draw a picture to illustrate each of the following properties.

1. For any real number $c$, we have $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$.

2. If $f(x) \geq 0$ on the interval $[a, b]$, then we have $\int_a^b f(x)dx \geq 0$.

3. If $f(x) \geq g(x)$ on the interval $[a, b]$, then we have $\int_a^b f(x)dx \geq \int_a^b g(x)dx$.

4. If $m \leq f(x) \leq M$ on the interval $[a, b]$, then we have $m(b - a) \leq \int_a^b f(x)dx \leq M(b - a)$.

## II. Indefinite Integrals/Anti-Derivatives

Without the bounds $a$ and $b$, what does the symbol $\int f(x)dx$ mean?

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**Exercise:** Just by thinking backward, find the following indefinite integrals.

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|---|---|---|---|---|---|
| 1. | $\int 10dx =$ | 6. | $\int \cos x dx =$ | 11. | $\int x^{1/3}dx =$ |
| 2. | $\int 2xdx =$ | 7. | $\int \sec^2 x dx =$ | 12. | $\int \cos(3x)dx =$ |
| 3. | $\int \frac{1}{x}dx =$ | 8. | $\int x^2 dx =$ | 13. | $\int e^{4x}dx =$ |
| 4. | $\int e^x dx =$ | 9. | $\int x^5 dx =$ | 14. | $\int (5x-1)^2 dx =$ |
| 5. | $\int \sin x dx =$ | 10. | $\int \sqrt{x}dx =$ | *15. | $\int \cos^2(x)\sin(x)dx =$ |

Did you forget the constant $C$?

16. Suppose you know that $F(1) = 2$ and $F'(x) = 2x$. Can you find $F(x)$?

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## III. Fundamental Theorem of Calculus

### A. Part 2 (c.f. Net Change Theorem)

Statement of the theorem:

$$\int_a^b f(x) =$$

Key: This gives us a much easier way to compute definite integrals (instead of using limits).

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### B. Part 1

Statement of the theorem: If $g(x) = \int_a^x f(t)dt$, then

Key: This says that integration and differentiation and inverse processes of each other.

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**Exercise:**

1. Let $g(x) = \int_3^x \sin(t)dt$. Find $g'(x)$.
2. Let $g(x) = \int_0^{x^2} \frac{u}{u+1}du$. Find $g'(x)$.
3. Let $g(x) = \int_{-x}^x \sqrt{1+t^2}dt$. Find $g'(x)$.

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