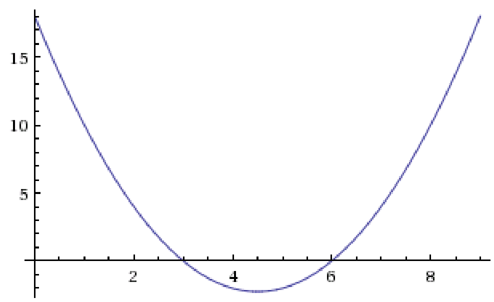
MATH 3B TRIGONOMETRIC INTEGRALS

I. Distance vs Displacement

Distance:

Displacement:

Example. Suppose a particle is moving along the x-axis with velocity $v(t) = t^2 - 9t + 18$.



a) Find the displacement of the particle from $t = 1$ to $t = 8$.

b) Find the total distance traveled by the particle from $t = 1$ to $t = 8$.

II. Trigonometric Integrals (Same Angle)

Main identities involved in this technique:

sin/cos (both even)	**sin/cos (one odd)**	**sec/tan (sec even)**	**sec/tan (tan odd)**

A. Sine and Cosine: Both are even

Main idea:

Example. Find $\int \cos^2 \theta d\theta$.

Example. Find $\int \cos^4 \theta d\theta$.

Example. Find $\int \sin^2 \theta \cos^2 \theta d\theta$.

B. Sine and Cosine: At least one is odd

Main idea:

Example. Find $\int \cos^2 \theta \sin \theta d\theta$.

Example. Find $\int \cos^3\theta \sin^2\theta d\theta$.

Example. Find $\int \cos^3\theta \sin^3\theta d\theta$.

As an exercise, do the secant/tangent case yourself. What is the main idea? What identities are involved? Start by considering specific examples (e.g. $\int \sec^4\theta \tan\theta d\theta$, $\int \tan^3\theta \sec^2\theta d\theta$). Consult the textbook (section 7.2) if you need.

C. Secant and Tangent: Secant is even

Main idea:

D. Secant and Tangent: Tangent is odd and there is at least one secant

Main idea:

III. Trigonometric Integrals (Different Angles)

Main identities involved in this technique:

Example. Find $\int \sin(3\theta)\cos(2\theta)d\theta$.

IV. Practice + Other Techniques

Problem1: Standard straight forward trig integrals
Problem2: Additional techniques on secant/tangent integrals
Problem3: Trig integrals with other techniques

You can check your answers using wolframalpha.

---

1. Use the techniques above to find the following indefinite integrals.

a) $\int \sin^2 x dx$
b) $\int \sin^3 u \cos^2 u du$
c) $\int \sin^2 x \cos^2 x dx$ (Note: for practice, try to do this without using the double angle formula)
d) $\int \frac{\cos^5 y}{\sqrt{\sin y}} dy$ (Note: we don't need both exponents to be integers)
e) $\int (1 + \cos x)^2$
f) $\int \sec^2 \theta \tan^2 \theta d\theta$
g) $\int \sec^4 t \tan^5 t dt$
h) $\int \sec \theta \tan^3 \theta d\theta$
i) $\int \sec^6 \theta \tan^5 \theta$
j) $\int \sin 5x \sin x dx$
k) $\int \sin 4t \cos 2t dt$

2. In the secant/tangent technique, some cases are not covered (e.g. when secant has odd power). You will need to use other techniques in these cases. The following formulae are helpful.

$$\int \tan x = \ln|\sec x| + C \text{ and } \int \sec x dx = \ln|\sec x + \tan x| + C.$$

Use these and techniques that you know to find the following indefinite integrals.

a) $\int \sec^3 x dx$ (Hint: integration by parts)
b) $\int \tan^2 x dx$ (Hint: rewrite this to an integral with an even secant power)
c) $\int \tan^4 x dx$ (Hint: a similar trick as part b)
d) $\int \tan^3 x dx$ (Hint: a similar trick as part b and then a little bit more work)

3. Sometimes the integrals will not be very straight forward. You might need to first simplify the integral, or use other techniques to compute them. Use u-sub or integration by parts if you need, find the following indefinite integrals.

a) $\int \frac{\sin^3(\sqrt{x})}{\sqrt{x}} dx$
b) $\int \theta \cos^2 \theta d\theta$
c) $\int \cos x \cos^5(\sin x) dx$
d) $\int \frac{\cos x + \sin 2x}{\sin x} dx$ (Hint: first use a double angle formula to make all of the angles the same)
e) $\int \frac{1 - \tan^2 x}{\sec^2 x} dx$ (Hint: simplify this integral)
f) $\int \frac{\tan^3 \theta}{\cos^4 \theta} d\theta$ (Hint: first try to write everything in terms of sine and cosine)
*g) $\int \frac{dx}{1 - \sin x}$ (Hint: this requires some algebraic manipulation before integrating)
