Math 3C Exam 2 Review Problems Part 2

Different models and their solutions:

	IVP	Solution	Note
Growth/Decay 1	$\frac{dy}{dt} = ky,\ y(0) = y_0$	$y(t) = y_0 e^{kt}$	
Growth/Decay 2	$\frac{dy}{dt} = ky + M,\ y(0) = y_0$	$y(t) = (y_0 + \frac{M}{k})e^{kt} - \frac{M}{k}$	$M =$ any constant
Cooling	$\frac{dT}{dt} = k(M - T),\ T(0) = T_0$	$T(t) = T_0 e^{-kt} + M(1 - e^{-kt})$	$M =$ room temp.
Logistic	$\frac{dy}{dt} = r(1 - \frac{y}{L})y,\ y(0) = y_0$	$y(t) = \frac{L}{1+(\frac{L}{y_0}-1)e^{-rt}}$	
Threshold	$\frac{dy}{dt} = -r(1 - \frac{y}{T})y,\ y(0) = y_0$	$y(t) = \frac{T}{1+(\frac{T}{y_0}-1)e^{rt}}$	

2.1 Linear Equations: The Nature of Their Solutions

Keywords: linear operators, linear DEs, order, homogeneous, constant/variable coefficients, Superposition Principle for Linear Homogeneous Equations, Nonhomogeneous Principle

1. **(Verifying the Superposition Principle)** Consider the homogeneous LDE

$$y'' - y' - 6y = 0.$$

a) Verify that $y_1 = e^{3t}$ and $y_2 = e^{-2t}$ are solutions.
b) Verify that $c_1 y_1 + c_2 y_2$ is also a solution for any real numbers c_1 and c_2.

2. **(Using the Nonhomogeneous Priniciple)** Consider the non-homogeneous LDE

$$y' - y = 3e^t.$$

a) Verify that $y_p(t) = 3te^t$ is a solution.
b) Find a solution to the homogeneous DE $y' - y = 0$.
c) Write down the general solution to the original DE.

(See last handout for more practice problems.)

2.2 Solving the First-Order Linear Differential Equation

Keywords: undetermined coefficients, integrating factors

(See last handout for practice problems.)

2.3 Growth and Decay Phenomena

3. **(Decay)** Radium decays at a rate proportional to the amount present and has a half-life of 1600 years. What percentage of an original amount will be present after 6400 years?

4. **(Growth)** If the number of bacteria in a culture is 5 million at the end of 6 hours and 8 million at the end of 9 hours, how many were present initially?

5. **(Compound interest with withdrawals)** Suppose Linda deposits one million dollars in a savings account that pays an annual rate of 8% compounded continuously. How long will the money last if she makes annual withdrawals of $100,000?

2.4 Linear Models: Mixing and Cooling

6. **(Mixing - HW)** Into a tank containing 100 gal of fresh water, Wei Chen was to have added 10 lb of salt but accidentally added 20 lb instead. To correct her mistake she started adding fresh water at a rate of 3gal/min, while drawing off well-mixed solution at the same rate. How long will it take until the tank contains the correct amount of salt?

7. **(Mixing)** A tank initially contains 200 gallons of fresh water, but then a salt solution of unknown concentration is poured into the tank at 2gal/min. The well-stirred mixture flows out of the tank at the same rate. After 120min, the concentration of salt in the tank is 1.4lb/gal. What is the concentration of the entering brine?

8. **(Cooling - HW)** In a murder investigation, a corpse was found by a detective at exactly 8 PM Being alert, he measures the temperature of the body and finds it to be 70°F. Two hours later the detective again measures the temperature of the corpse and finds it to be 60°F. If the room temperature is 50°F, and assuming the body temperature of the person before death was 98.6°F, how many hours before 8 PM did the murder occur?

9. **(Cooling)** Professor Farlow always has a cup of coffee before his $8:00$ AM class. Suppose the coffee is 200°F when poured from the coffee pot at $7:30$AM, and 15 minutes later it cools to 120°F in a room whose temperature is 70°F. However, Professor Farlow never drinks his coffee until it cools to 90°F. When will the professor be able to drink his coffee?

2.5 Nonlinear Models: Logistic Equation

Keywords: equilibrium, phase line, stable/sink, unstable/source, semistable/node, logistic equation, threshold equation

10. **(Equilibrium solutions - HW)** Consider the differential equation $y' = y - \sqrt{y}$.
a) Find all equilibrium solutions.
b) Use phase line to classify the stability of each of the equilibrium solution.

11. **(Logistic equation)** Suppose that we start at time $t_0 = 0$ with a sample of 1000 cells. One day later we see that the population has doubled, and some time later we notice that the population has stabilized at $100,000$. Assume a logistic growth model.
a) What is the population after 5 days?
b) How long does it take the population to reach $50,000$ cells?