MATH 3C INTEGRATION AND DIFFERENTIAL EQUATIONS

I. Integration methods

1. What integration methods do you know?

2. What method(s) would you use to integrate the following integrals?

a) $\int \frac{x^2+1}{x^3+3x-10} dx$

b) $\int x \sin(2x) dx$

c) $\int \frac{2x}{x^2+3x+2} dx$

d) $\int \sec^3 x \tan x \, dx$

e) $\int \cos(x^2) dx$

3. Now actually solve the integrals.

II. Differential equation

A differential equation is an equation that involves a function and its derivatives, e.g. $y' + 2y = 1$. The unknown in a differential equation is the function y.

1. Solution to a differential equation:

a) $9y'' + 4y = 0$. Is $y = 2\cos(\frac{2}{3}t)$ a solution?

(Hint: If you have no idea what to do, consider this analogy: if I ask you whether $(2, 1)$ is a solution to the equation $y = x - 1$, what would you do?)

b) $y'' = -25y$. Is $y = \sin(5t)$ a solution?

c) $y'' - 2y' + y = 0$
Is $y = 2e^t - te^t$ a solution?

2. Solving a differential equation:

How would you solve this? You might not have learned this but just try something.

$$\frac{dy}{dt} = \frac{t}{y}, \quad y(0) = 1.$$

III. Extra practice

If you are rusty on integrals here are more problems you can do.

1. $\int \frac{x^4}{\sqrt{2x^5 - x^{10}}} dx$

2. $\int \frac{1}{x^3 - 4x^2 + 5x} dx$

3. $\int \frac{e^{2\sqrt{x}}}{\sqrt{x}} dx$

4. $\int \sin^3 x \, dx$

5. $\int x(x-2)^{\frac{1}{3}} dx$

6. $\int x^2 e^{-x^3} dx$

7. $\int \frac{\ln x}{x^4} dx$

8. $\int x^2 e^{3x} dx$

9. $\int x\sqrt{x+3} \, dx$

10. $\int \cos^2(2x) dx$

11. $\int \frac{e^x}{e^{2x} + 2e^x + 1} dx$

12. $\int \frac{2x-1}{x^2 - 3x + 2} dx$

13. $\int \sec^4(3x) dx$

14. $\int \cos^2(2x) \sin^3(2x) dx$

15. $\int \frac{\sqrt{x^2 - 9}}{x} dx$

16. $\int \frac{x^2}{(x-2)^4} dx$

17. $\int e^x \sin x \, dx$