Math 3C Separation of Variables and Euler's Method

I. a) Finding equilibrium solutions

Recall: What is an equilibrium solution?

b) Characterizing equilibrium solutions

Recall: When is an equilibrium solution stable/unstable/semi-stable?

Examples.

1. Find and characterize all, if any, equilibrium solutions to the DE $y' = y + t$.

2. Find and characterize all, if any, equilibrium solutions to the DE $y' = y^2 + 5y + 4$.

II. Separation of variables

What kind of method is this? How does it work? When do you use it?

Examples.

1. Solve the IVP $y' = (y - 1)(3 - y)$, $y(0) = 2$.

2. Find the general solution to the DE $y' = y \sin t$.

III. Euler's method

What kind of method is this? How does it work?

Example.

Data: $y' = 3t^2 - y$, $y(0) = 1$.

Task: a) Use Euler's method to approximate the solution to this IVP on the interval $[0, 5]$ with step size 1.

b) Use part a) to approximate $y(1.5)$.

IV. Practice

1. Find and characterize all equilibrium solutions to the DE $y' = (y - 1)(y^2 + 3y + 2)$.
2. Find and characterize all equilibrium solutions to the DE $y' = y^2 t$.
(Hint: What is the sign of y' when $t \to \infty$?)
3. Find the general solution to the DE $y' = (t^2 + 1)y$.
(Hint: Remember $\int \frac{1}{y} dy = \ln |y| + C$. How do you get rid of the the absolute value sign?)
4. Find the general solution to the DE $y' = (y^2 + 5y + 6)(t^2 - 4)$.
5. Solve the IVP $y' = \frac{1}{y(1+t^2)}$, $y(0) = -1$.
(Hint: Remember when you have $y^2 = (\cdots)$, you get $y = \pm\sqrt{(\cdots)}$. How do you decide whether to take + or −?)
6. Solve the IVP $y' = y^2 t e^{-t}$, $y(0) = 1$.
7. Solve the IVP $y' = \frac{\sqrt{1+3t}}{\sin y}$, $y(0) = \pi$. Leave your answer as an implicit equation relating y and t, i.e. you do not have to solve for y explicitly.
8. Use Euler's method to approximate the solution to the IVP $y' = t^2 - e^y$, $y(0) = 0$ on the interval $[0, 5]$ with step size 1.
9. a) Use Euler's method to approximate the solution to the IVP $y' = \frac{-t}{y}$, $y(0) = 1$ on the interval $[0, 2]$ with step size 0.5.
b) Use your answer in part a) to estimate $y(1.5)$.
c) Use separation of variables to solve the same IVP.
d) Use your answer in part c) to find $y(1.5)$. Compare this to the approximation you got in part b). How good was your approximation?