Math 3C Matrix Operations

I. Matrix

$$(a_{ij}) = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & & & \vdots \\ \vdots & & & \vdots \\ \vdots & & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

II. Matrix operations

$$A = \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 4 \\ 3 & 1 \end{bmatrix} \quad D = \begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix} \quad E = \begin{bmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{bmatrix}$$

1. Addition

2. Multiplication (by scalar)

3. Multiplication

4. Transpose

III. Practice and some conceptual questions

1. a) Compute $2D + E$ and $3(B - 2C)$.

b) Given two matrices P and Q, what must their dimensions satisfy for you to be able to add them? Then determine which of the following expressions make sense.

$$B + 5C,\ D - E,\ A + 2B$$

2. a) Compute AB and DA.

b) Given two matrices P and Q, what must their dimensions satisfy for you to be able to multiply them? Then determine which of the following expressions make sense.

$$CE,\ BA,\ A(B + C),\ BA^T$$

3. a) Let $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. Compute I_2B, BI_2, I_2C, and CI_2.

b) Given any 2×2 matrix P, what can you say about PI_2 and I_2P?

(This matrix I_2 is called the **identity matrix** (of size 2). Notice 1 has the property that $1 \cdot r = r = r \cdot 1$ for any real number r. The identity matrix I_2 has the same property among all 2×2 matrices.)

c) What would be the identity matrix I_n (of size n) among all $n \times n$ matrices? It should be a matrix such that $I_nQ = Q = QI_n$ for all $n \times n$ matrices Q.

4. a) Compute BC and CB.

b) An operation is said to be **commutative** if order does not matter. For example, multiplication of real numbers is commutative, because we can multiply numbers in any order we want and still get the same answer (e.g. $2 \cdot 3 = 3 \cdot 2$). What about multiplication of matrices? Is it a commutative operation?

5. a) Compute $(AB)C$ (multiply A and B first and then multiply the result by C) and $A(BC)$ (multiply B and C first and then multiply the result by A).

b) You should find that $(AB)C = A(BC)$. This is actually true for any matrices. We say that matrix multiplication is **associative**, which means that we can put parentheses in any way we want when computing and still get the same answer. Notice that multiplication of real numbers is also associative (e.g. $(2 \cdot 3) \cdot 9 = 2 \cdot (3 \cdot 9)$).