Math 3C Matrix Operations & System of Equations

I. Vocabulary

1. Order of a matrix
2. Column/row vector
3. Zero matrix
4. Diagonal matrix
5. Identity matrix
6. Scalar product
7. Orthogonal
8. Length of a vector
9. Augmented matrix
10. RREF
11. Inconsistent system
12. Consisten system
13. Underdetermined system
14. Rank of a matrix

II. System of equations

Step1: Write the system of equations in its augmented matrix form.

Step2: Use Gauss-Jordan reduction to reduce the matrix to RREF.

Step3: Write down the solution based on the RREF (if the system is consistent).

Example.

$$\begin{aligned} x + y + 2z &= 1 \\ 2x - y + z &= 2 \\ 4x + y + 5z &= 4 \end{aligned}$$

1.

$$\begin{aligned} x + 2y &= 1 \\ 2x - y &= 0 \\ 3x + 2y &= 1 \end{aligned}$$

2.

$$\begin{aligned} x + y + z &= 0 \\ y + z &= 1 \end{aligned}$$

III. True/false

A. Matrix Operations and orders

1. An $m \times n$ matrix has m columns and n rows.

2. Addition of two matrices is only possible if they have the same order.

3. Multiplication of two matrices is only possible they have the same order.

4. Multiplying a matrix by a scalar does not change its order.

5. Let A be an $m \times n$ matrix and B be an $n \times p$ matrix. The product AB is well-defined and has order $m \times p$.

6. Let A and B be two $n \times n$ matrices. It is always true that $AB = BA$.

7. Let I_n be the $n \times n$ identity matrix. The matrix cI_n is diagonal for any real number c.

8. For any matrix A, the identity $(A^T)^T = A$ holds.

9. Let A be a square matrix. The order of A and A^T are the same.

10. Let $\mathbf{x}$ and $\mathbf{y}$ be two vectors. The scalar product $\mathbf{x} \cdot \mathbf{y}$ is another vector.

11. The vectors $\mathbf{x} = [1\ 2\ -1]$ and $\mathbf{y} = [0\ 1\ 2]$ are orthogonal in $\mathbb{R}^3$.

12. The vector $\mathbf{x} = \frac{1}{\sqrt{5}}[2\ 1\ -1]$ is a unit vector in $\mathbb{R}^3$.

13. A symmetric matrix must be square.

B. System of equations

14. The matrix $\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is in RREF.

15. The matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is in RREF.

16. The rank of the matrix $\begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ is 2.

17. Let $[\mathbf{A}|\mathbf{b}]$ represents consistent system of equations. If the rank of A is equal to the number of variables, then the system has a unique solution.

18. A system of three equations and three unknowns always has a unique solution.

19. A consistent system with more variables than equations must have infinitely many solutons.

20. A homogeneous system $\mathbf{Ax} = \mathbf{0}$ is always consistent.