Math 3C Vector spaces, Subspaces, Basis, and Dimension

I. Vector Spaces

⋄Some familiar vector spaces⋄

1. Euclidean spaces: $\mathbb{R}^2$, $\mathbb{R}^3$, $\mathbb{R}^n$
2. Polynomial spaces: $\mathbb{P}$ (all polynomials), $\mathbb{P}_n$ (all polynomials of degree $\leq n$)
3. Matrix spaces: $\mathbb{M}_{mn}$ (all $m \times n$ matrices)
4. Function spaces: $C(I)$ (all continuous functions on the interval I)

II. Subspaces

Definition. A subspace of a vector space $\mathbb{V}$ is a subset $\mathbb{W}$ that is itself a vector space.

Vector Subspace Theorem. A nonempty subset $\mathbb{W}$ of a vector space $\mathbb{V}$ is a subspace of $\mathbb{V}$ if

1.

2.

How to determine if a subset is a subspace or not:

1. (cf. HW10 #12) $\mathbb{V} = \mathbb{P}_3$ and $\mathbb{W} = \{p(t) | p(0) = 0\}$

2. (HW10 #3) $\mathbb{V} = \mathbb{M}_{22}$ and $\mathbb{W} = \{\mathbf{A} | \mathbf{A} \text{ is diagonal}\}$

3. (HW10 #6) $\mathbb{V} = \mathbb{R}$ and $\mathbb{W} = \mathbb{Z}$

III. Basis and dimension

Definition. The set $\{\mathbf{v}_1, \mathbf{v}_2, ..., \mathbf{v}_n\}$ is a **basis** for a vector space $\mathbb{V}$ if

1.

2.

Definition. The **dimension** of $\mathbb{V}$ is defined to be

⋄Standard basis for some familiar spaces⋄

1. $\mathbb{R}^3$

2. $\mathbb{M}_{23}$

3. $\mathbb{P}_2$

How to determine if a given set of vectors forms a basis or not:

1. Check the number of vectors in the set.

a) (cf. HW10 #16) $\mathbb{V} = \mathbb{M}_{22}$

$$S = \{ \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 3 & 1 \\ 0 & 4 \end{bmatrix} \}$$

b) $\mathbb{V} = \mathbb{R}^3$

$$S = \{ \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ -5 \\ 3 \end{bmatrix} \}$$

2. Check if the vectors are linear independent.

a) (cf. HW10 #17) $\mathbb{V} = \mathbb{R}^3$

$$S = \{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix} \}$$

b) (cf. HW10 #22) $\mathbb{V} = \mathbb{P}_2$

$$S = \{3, 2t - 1, t^2 + t + 1\}$$