MATH 4A FALL 2015 MIDTERM 2 REVIEW PART I

Name: Section: 8AM 5PM 6PM 7PM

*This review worksheet is due at the beginning of section next Monday (Nov 2). Each answer is worth 1 point. You need 11 points (out of 17 possible points) to receive 1% on your discussion section grade.

Linear Transformation

1. Let $T : \mathbb{R}^n \longrightarrow \mathbb{R}^m$ be defined by $T(\mathbf{x}) = A\mathbf{x}$, where A is the matrix below.

$$A = \begin{bmatrix} 2 & -1 & 0 & 3 \\ 1 & 1 & 2 & -1 \end{bmatrix}$$

a) What is n?

b) What is m?

c) Determine whether T is (i) onto and (ii) one-to-one. Justify your answers.

2. Let $T : \mathbb{R}^3 \longrightarrow \mathbb{R}^2$ be a linear transformation such that

$$T\left(\begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \qquad T\left(\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} -1 \\ 3 \end{bmatrix} \qquad T\left(\begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 5 \end{bmatrix}.$$

a) Find the matrix of T.

b) Compute $T(1, 1, 1)$.

3. Let $T : \mathbb{R}^n \longrightarrow \mathbb{R}^m$ be a linear transformation.

a) If $m > n$ then T must be (i) onto (ii) one-to-one (circle all possibilities).

b) If $m < n$ then T must be (i) onto (ii) one-to-one (circle all possibilities).

c) If $m = n$ then T must be (i) onto (ii) one-to-one (circle all possibilities).

d) If $m = n$ and T is onto, then T is also one-to-one. True/False? Explain why it is true or give a counterexample.

Matrix Multiplication

1. Compute the following. Show your work.

$$\begin{bmatrix} 0 & 4 \\ 2 & -1 \\ 1 & 0 \end{bmatrix}^T \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} =$$

---

2. a) Compute (1 point for all three answers)

$$\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}^2 =$$

$$\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}^3 =$$

$$\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}^4 =$$

b) Do you notice any pattern? If yes, write down

$$\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}^{1000} =$$

3. Let I_2 and $\mathbf{0}_2$ be the 2×2 identity and zero matrices, respectively.

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad \mathbf{0}_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Give examples of 2×2 matrices A and B satisfying the given property. Show that your examples really work!
[The following examples show that multiplication of matrices is very different from that of real numbers!!]
a) $AB \neq BA$. [But for $a, b \in \mathbb{R}$, we have $ab = ba$.]

b) $AB = \mathbf{0}_2$ but neither A nor B is equal to $\mathbf{0}_2$. [But for $a, b \in \mathbb{R}$, we have $ab = 0$ if and only if $a = 0$ or $b = 0$.]

c) $A^2 = I_2$ but $A \neq \pm I_2$. [But for $a, b \in \mathbb{R}$, we have $a^2 = 1$ if and only if $a = \pm 1$.]
