MATH 4A FALL 2015 WORKSHEET 4

Name: Section: 8AM 5PM 6PM 7PM

*Each answer is worth 1 point. You need 6 points to get 1% on your discussion section grade.

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1. Complete the definition: A set $\{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$ of vectors in $\mathbb{R}^n$ is said be *linearly independent* if

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2. True of False. No justification is required.

a) The columns of a matrix A are linearly independent if and only if the equation $A\mathbf{x} = \mathbf{0}$ has a unique solution. True False

b) The columns of a 3×5 matrix are always linearly depedent. True False

c) The columns of a 5×3 matrix are always linearly indepedent. True False

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3. Find the value(s) of h for which the vectors below are linearly independent.

$$\begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix} \qquad \begin{pmatrix} -1 \\ 7 \\ 1 \end{pmatrix} \qquad \begin{pmatrix} 3 \\ 8 \\ h \end{pmatrix}$$

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4. Let A be a 3×3 matrix whose columns are linearly independent.

a) What is the reduced echelon form of A?

b) Let $\mathbf{b}$ be a vector in $\mathbb{R}^3$. The equation $A\mathbf{x} = \mathbf{b}$ has (circle all possibilities)

(i) no solution (ii) a unique solution (iii) infinitely many solutions

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5. Let A be a 3×3 matrix and let $\mathbf{b}$ be a vector in $\mathbb{R}^3$.

a) If there exist two different vectors $\mathbf{x}_1$ and $\mathbf{x}_2$ in $\mathbb{R}^3$ such that $A\mathbf{x}_1 = \mathbf{b}$ and $A\mathbf{x}_2 = \mathbf{b}$, then the columns of A are (circle all possibilities)

(i) linearly indepedent (ii) linearly dependent

b) If the equation $A\mathbf{x} = \mathbf{b}$ has no solution, then the columns of A are (circle all possibilities)

(i) linearly indepedent (ii) linearly dependent

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6. The following statements are all false. Explain why or give a counterexample.

a) A set $\{\mathbf{v}_1\}$ containing only one vector is always linearly indepedent.

b) If $\{\mathbf{v}_1, \mathbf{v}_2\}$ is linearly dependent, then $\mathbf{v}_2$ is a scalar multiple of $\mathbf{v}_1$.

c) If $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is linearly dependent, then $\mathbf{v}_3 \in \mathrm{Span}(\mathbf{v}_1, \mathbf{v}_2)$.

d) If $A\mathbf{x} = \mathbf{0}$ has the trivial solution, then the columns of A are linearly indepedent.

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7. This problem will not be graded.

$$A = \begin{pmatrix} 1 & 3 & 2 & -6 & 0 & 0 \\ 0 & 0 & 1 & -7 & 4 & -8 \\ 0 & 0 & 0 & 0 & 2 & 6 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \qquad \mathbf{x}_0 = \begin{pmatrix} 2 \\ -1 \\ 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} \qquad \mathbf{b} = \begin{pmatrix} -7 \\ -3 \\ 2 \\ 0 \end{pmatrix}$$

a) Describe all solutions of $A\mathbf{x} = \mathbf{0}$ in parametric vector form.

b) Verify that $A\mathbf{x}_0 = \mathbf{b}$.

c) Describe all solutions of $A\mathbf{x} = \mathbf{b}$ in parametric vector form. [Hint: Use parts (a) and (b).]

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8. This problem will not be graded. Determine if the sets of vectors below span $\mathbb{R}^3$.

a) $\left\{ \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix}, \begin{pmatrix} 3 \\ 0 \\ -5 \end{pmatrix} \right\}$ b) $\left\{ \begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 6 \\ 0 \end{pmatrix} \right\}$ c) $\left\{ \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} -1 \\ 4 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 4 \\ 1 \end{pmatrix} \right\}$

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9. This problem will not be graded. Determine if the set of vectors below is linearly independent.

$$\left\{ \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 4 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix} \right\}$$

If not, find a non-trivial relation among the vectors (i.e. write $\mathbf{0}$ as a linear combination of the vectors so that at least one of the weights is non-zero).