1. Let $T : \mathbb{R}^3 \longrightarrow \mathbb{R}^2$ be defined by

$$T(x_1, x_2, x_3) = (x_1 + 2x_2, x_1 - 4x_2).$$

Write down the standard matrix of T.

2. Consider the matrix

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & -2 & -4 \\ 1 & 3 & 5 \\ 4 & 0 & 1 \end{bmatrix}$$

and let T be the linear transformation defined by $T(\mathbf{x}) = A\mathbf{x}$.

a) Find the domain and codomain of T.

$$\text{Domain}(T) =$$
$$\text{Codomain}(T) =$$

b) Is T onto?

                Yes   No

3. Let $T : \mathbb{R}^5 \longrightarrow \mathbb{R}^2$ be a linear transformation.

a) T cannot be one-to-one.

                True   False

b) T must be onto.

               True   False

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4. Let $T : \mathbb{R}^4 \longrightarrow \mathbb{R}^4$ be a linear transformation and A its standard matrix.

a) If the columns of A are linearly independent, then T is onto.

               True   False

b) If T is onto, then T is also one-to-one.

               True   False

c) If T is one-to-one, then $T(\mathbf{x}) = \mathbf{b}$ has a unique solution for all $\mathbf{b}$ in $\mathbb{R}^4$.

               True   False

                                      Score: /10