MATH 8 WINTER 2015 HOW TO WRITE PROOFS

A **mathematical proof** is an argument which convinces other people that something is true.

- A proof should be written in complete sentences.
- Never start a proof with the desired conclusion.

- **State your strategy**: Begin your proof by explaining the general line of reasoning. If you are going to use induction or proof by contradiction, tell your reader at the beginning of the proof.
- **Keep a linear and logical flow**: The steps of your argument should follow one another in a smooth and sequential way. Use conjunctions, such as *hence* and *because*, to connect the steps logically.
- **Explain your reasoning**: Justify any claims that you make in your argument. Does it follow from the definition, the hypothesis, or something that you proved earlier in the argument?
- **Avoid excessive symbolism**: Do not use logical symbols such as $\forall$, $\exists$, $\neg$, or $\therefore$ in your argument.
- **Simplify**: Be concise. Do not make your proof unnecessarily long and complicated. It is not true that the more your write the better.
- **Introduct notation thoughtfully**: Introduce a variable or notation if it makes the argument easier to follow. Be sure to define the meanings of any notation that you introduce.
- **Structure long proofs**: Use multiple paragraphs if necessary. If it makes the argument simpler, consider proving facts that you need as preliminary lemmas, and then cite them in your proof.
- **Conclude your proof**: Always tell you reader when you have completed the proof. If it is not clear, explain why the original claim follows from your argument.

Be critical when you read a proof.

Claim 1. Let x be a real number. If $x(2x+1) \geq 3x$ then $x \geq 1$.

If $x(2x+1) \geq 3x$, then dividing both sides by x yields

$$\begin{aligned} 2x + 1 &\geq 3 \\ 2x &\geq 2 \\ x &\geq 1, \end{aligned}$$

which shows that indeed $x \geq 1$.

What's wrong in the above?

Comment on the following proofs. How can you make them better?

Claim2. The sum of any two odd natural numbers is even.

$$x = 2n + 1 \qquad\qquad y = 2m + 1$$

$$x + y = 2n + 1 + 2m + 1 = 2(n + m + 1) \qquad \checkmark$$

How can you make it better?

Claim3. Show that $x^2 + y^2 \geq 2xy$ for any real numbers x and y.

$$\begin{aligned} x^2 + y^2 &\geq 2xy \\ x^2 - 2xy + y^2 &\geq 0 \\ (x-y)^2 &\geq 0 \qquad \checkmark \end{aligned}$$

How can you make it better?

Now try to write up a proof of the following statement.

Claim4. The product of any three consecutive natural numbers is always divisible by four.