LINEAR ALGEBRA

- Matrix Terminology & Corresponding Properties: Dimension (Order), Element (Entry), Row Vector, Column Vector, Square Matrix, Diagonal Element, Augmented Matrix, Reduced Row Echelon Form (RREF), Inverse Matrix

- Special Matrix & Corresponding Properties: Zero Matrix, Diagonal Matrix, Identity Matrix

- Operation for Single Matrix & Corresponding Properties: Scalar Multiplication, Determinant, Elementary Row (Column) Operations,

- Matrix Arithmetic (Between Matrices) & Corresponding Properties: Addition, Subtraction, Multiplication, Division

- Be Able to Solve Systems of Linear (Algebraic) Equations by Gaussian Elimination for Different Cases: Unique Solution, Infinitely Many Solutions, No Solution; Understand Correspondence Between Pivot, Rank, Invertibility, Singularity & Number of Solutions; Be Able to Solve Linear Systems by Nonhomogeneous Principle & Cramer’s Rule

- Vector Space (Ten Conditions), Subspace (Two Conditions) & Their Characterization: Basis & Dimension

PRACTICE PROBLEMS

1

\[ A = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & -2 \\ 0 & 1 & 1 \end{pmatrix}, \quad C = \begin{pmatrix} 3 & 1 & -1 \\ -1 & 1 & 3 \\ 1 & -1 & 1 \end{pmatrix}, \]

and \( k = 1/4 \), calculate \( B = kC, A + C, AB, BA, |A|, |AB|, |BA|, A^{-1}. \)
2 Solve the following linear systems by Gaussian elimination, nonhomogeneous principle, and Cramer’s rule (if possible)

\[
\begin{align*}
x + z &= 2 \\
2x - 3y + 5z &= 4 \\
3x + 2y - z &= 4
\end{align*}
\]

, \n
\[
\begin{align*}
x + 2y + z &= 2 \\
2x - 4y - 3z &= 0 \\
-x + 6y - 4z &= 2 \\
x - y &= 4
\end{align*}
\]

, \n
\[
\begin{align*}
x_1 + 2x_3 - 4x_4 &= 1 \\
x_2 + x_3 - 3x_4 &= 2
\end{align*}
\]

3 Show that the set \( C^1(\mathbb{R}) = \{ \text{all continuously differentiable functions over } \mathbb{R} \} \) is a vector space. Show that the following sets:

\[
V = \{ y(t) | y(t) \in C^1(\mathbb{R}), \text{ and } y(1) = 0 \},
\]

and

\[
W = \{ y(t) | y(t) \in C^1(\mathbb{R}), \text{ and } y' + ty = 0 \}
\]

are subspaces of \( C^1(\mathbb{R}) \).

4 Show that \( S = \{ [1, 0, 0], [1, 1, 0], [1, 1, 1] \} \) forms a basis of \( \mathbb{R}^3 \).