Math 5C Spring 2010
Exam 1

April 16, 2010

M. Choice	
F. Resp. 1	
F. Resp. 2	
F. Resp. 3	
F. Resp. 4	
Total	

Name______________________________

Perm No.______________________________

Directions:

1. There are 125 points on this exam; 100 points = 100%.
2. Each multiple choice problem is 5 points.
3. Each multiple choice problem has exactly one best answer.
4. No multiple choice problem requires heavy computation.
5. Each free response problem is 20 points.
6. Free response questions require justification; no work, no credit.
7. **A blank free-response problem is awarded 5 points.**
8. You are allowed one (1) 3 by 5 notecard.

Potentially useful integrals:

$$\int \ln t\,dt = t\ln t - t + C$$
$$\int t\ln t\,dt = \frac{1}{2}t^2\ln t - \frac{1}{4}t^2 + C$$

Multiple Choice

1. Let R be the region inside the cylinder $x^2+y^2=7$, above the plane $z=0$, and below the cone $z=\sqrt{x^2+y^2}$. In which coordinate system can R be described most simply?

   (a) Cartesian
   (b) Cylindrical
   (c) Spherical
   (d) Intergalactic planetary
   (e) Planetary intergalactic

2. Let D be the unit disk centered at $(0,1)$ in $\mathbb{R}^2$. What is $\iint_D (5x+2)\,dA$?

   (a) 0
   (b) 2π
   (c) 5π
   (d) 7π
   (e) None of the above

3. Let S be the sphere of radius R centered at the origin. What is $\iint_S (x^2+y^2+z^2)\,d\sigma$?

   (a) 0
   (b) $4\pi R^2$
   (c) $16\pi^2 R^4$
   (d) $4\pi R^4$
   (e) $16\pi^2 R^2$

4. Given a surface and a vector field, the flux is a measurement of what?

   (a) Amount of stuff crossing the surface
   (b) Amount of stuff flowing on the surface
   (c) Area
   (d) The enclosed volume
   (e) Fluxing

5. Let R be the unit cube in $\mathbb{R}^3$ given by $1 \le x,y,z \le 2$. Evaluate $\iiint_R dV$.

   (a) 1
   (b) 3
   (c) 7
   (d) 7/3
   (e) $V^3/6+C$

6. Let C be a unit circle in $\mathbb{R}^3$, oriented counterclockwise. Given $\mathbf{T}$ and $\mathbf{N}$, the unit tangent and unit normal vectors respectively, define the unit binormal vector $\mathbf{B} = \mathbf{T} \times \mathbf{N}$. Evaluate

$$\oint_C \mathbf{B} \cdot d\mathbf{r}$$

   (a) 0
   (b) π
   (c) 2π
   (d) -2π
   (e) None of the above

7. Which of these integrals gives the area of a disk D of radius 1?
   (a) $\iint_D x^2 + y^2 \, dA$
   (b) $\int_0^1 \int_0^{2\pi} d\theta \, dr$
   (c) $\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} dy \, dx$
   (d) $\iint_D r \, dA$
   (e) $\int_0^1 \int_0^{\sqrt{1-y^2}} 4 \, dx \, dy$

8. The function $\mathbf{\Phi}(\theta, \phi) = (\cos\theta \sin\phi, \sin\theta \sin\phi, \cos\phi)$ parametrizes which surface?
   (a) A cone
   (b) A cylinder
   (c) A sphere
   (d) A hyperbolic paraboloid
   (e) A rhombicosidodecahedron

9. (Bonus Problem) What is the answer to this question?
   (a) D
   (b) A
   (c) C
   (d) None of the above
   (e) E

Free Response

1. Let D be the unit disk in $\mathbb{R}^2$, centered at the origin. Evaluate

$$\iint_D \ln \sqrt{x^2 + y^2}\, dA.$$

2. Evaluate

$$\iiint_{\mathbb{R}^3} \exp[-(x^2+y^2+z^2)^{3/2}]\,dV.$$

Notation: $\exp(t)$ means e^t.

3. Let S be the surface in $\mathbb{R}^3$ given by $x^2 + y^2 = 1$, $0 \leq z \leq 1$, and $x \geq 0$. Assume that S is oriented with normal in the positive x direction. Find the flux of the vector field $\mathbf{F}(x, y, z) = x\mathbf{i}$ across S.

4. Let C be the (oriented) straight-line path in space from $(0, 0, 0)$ to $(1, 2, 3)$. Evaluate

$$\int_C yz\,dx + xz\,dy + e^x\,dz.$$