Math 5C Spring 2010
Exam 1 Solutions

April 16, 2010

M. Choice	
F. Resp. 1	
F. Resp. 2	
F. Resp. 3	
F. Resp. 4	
Total	

Name_____

Perm No._____

Directions:

1. There are 125 points on this exam; 100 points = 100%.
2. Each multiple choice problem is 5 points.
3. Each multiple choice problem has exactly one best answer.
4. No multiple choice problem requires heavy computation.
5. Each free response problem is 20 points.
6. Free response questions require justification; no work, no credit.
7. **A blank free-response problem is awarded 5 points.**
8. No notes, books, or electronic devices are allowed.

Potentially useful integrals:

$$\int \ln t \, dt = t \ln t - t + C$$

$$\int t \ln t \, dt = \frac{1}{2} t^2 \ln t - \frac{1}{4} t^2 + C$$

Multiple Choice

1. (B) The region in cylindrical coordinates is simply $0 \le \theta \le 2\pi$, $0 \le r \le \sqrt{7}$, $0 \le z \le r$.
2. (B) The average value of $5x + 2$ over the disk is clearly 2 by symmetry.
3. (D) $x^2 + y^2 + z^2 = R^2$ everywhere on S.
4. (A)
5. (A) $(2-1)(2-1)(2-1) = 1$
6. (A) $\mathbf{B}$ is orthogonal to $\mathbf{T}$ and hence $d\mathbf{r}$ at every point.
7. (E)
8. (C)
9. (C) Answers A and B contradict themselves, so the answer is C, D, or E. If the answer is not D, then it must *ONE* of the above, so either way E is not the answer. If the answer were D, A would have been correct, which is a contradiction. So D is wrong; the only remaining choice is C.

Free Response

1. In polar,

$$\iint_D \ln\sqrt{x^2+y^2}\,dA = \int_0^{2\pi}\int_0^1 r\ln r\,dr\,d\theta = 2\pi\int_0^1 r\ln r\,dr = \frac{\pi}{2}\left(2r^2\ln r - r^2\right)_0^1 = -\frac{\pi}{2}.$$

2. In spherical,

$$\begin{aligned}\iiint_{\mathbb{R}^3} \exp[-(x^2+y^2+z^2)^{3/2}]\,dV &= \int_0^{2\pi}\int_0^{\pi}\int_0^{\infty} \rho^2 e^{-\rho^3}\sin\phi\,d\rho\,d\phi\,d\theta \\ &= 4\pi\int_0^{\infty}\rho^2 e^{-\rho^3}\,d\rho \\ &= -\frac{4\pi}{3}e^{-\rho^3}\Big|_0^{\infty} \\ &= \frac{4\pi}{3}.\end{aligned}$$

3. From geometry, $d\sigma = 1\,d\theta\,dz$ and $d\mathbf{A} = \mathbf{n}d\sigma = (\cos\theta, \sin\theta, 0)\,d\theta\,dz$. The flux is

$$\iint_S (x,0,0)\cdot d\mathbf{A} = \int_0^1\int_{-\pi/2}^{\pi/2}\cos^2\theta\,d\theta\,dz = \frac{1}{2}\int_{-\pi/2}^{\pi/2}(1+\cos 2\theta)\,d\theta = \frac{\pi}{2}.$$

4. Parametrize: $x = t$, $y = 2t$, $z = 3t$, where $0 \le t \le 1$.

$$\int_C yz\,dx + xz\,dy + e^x\,dz = \int_0^1 (6t^2 + 6t^2 + 3e^t)\,dt = 1 + 3e.$$