Math 5C Spring 2010
Exam 2
Version A

May 7, 2010

M. Choice	
F. Resp. 1	
F. Resp. 2	
F. Resp. 3	
F. Resp. 4	
Total	

Name______________________________

Perm No.______________________________

Directions:

1. There are 125 points on this exam; 100 points = 100%.
2. Each multiple choice problem is 5 points.
3. Each multiple choice problem has exactly one best answer.
4. No multiple choice problem requires heavy computation.
5. Each free response problem is 20 points.
6. Free response questions require justification; no work, no credit.
7. **A blank free-response problem is awarded 5 points.**
8. You are allowed one (1) 3 by 5 notecard.
9. No other notes, books, or electronic devices are allowed.

Potentially useful formulas:

$$\nabla \cdot (f\mathbf{G}) = \nabla f \cdot \mathbf{G} + f\nabla \cdot \mathbf{G}$$
$$\nabla \times (f\mathbf{G}) = \nabla f \times \mathbf{G} + f\nabla \times \mathbf{G}$$

Multiple Choice

1. For which a is $(ax \ln y, 2y + x^2/y)$ the gradient of a function in (some subset of) $\mathbb{R}^2$?
   (a) 0
   (b) 1
   (c) 2
   (d) There is no such a
   (e) None of the above

2. Which of the following series diverges?
   (a) $\sum(-1)^n/(2n)$
   (b) $\sum 1/n^2$
   (c) $\sum 1/2^n$
   (d) $\sum 2/n$
   (e) $\sum 2^2/n^n$

3. Let R be a region in $\mathbb{R}^3$ with smooth boundary, oriented outward. Find the volume of R, given that
$$\iint_{\partial R} (5x, 3z, 4y) \cdot d\mathbf{A} = 120.$$
   (a) 2
   (b) 10
   (c) 24
   (d) 30
   (e) None of the above

4. For the vector field $\mathbf{F} = (y, 2z, 3x)$, what is the flux of $\nabla \times \mathbf{F}$ out of the unit ball centered at the origin in $\mathbb{R}^3$?
   (a) 0
   (b) 4π
   (c) $(4/3)\pi$
   (d) 8π
   (e) 12π

5. A smooth vector field $\mathbf{F}$ on $\mathbb{R}^3$ satisfies $\oint_C \mathbf{F} \cdot d\mathbf{r} = 0$ for every piecewise smooth closed curve C. Which of the following must be true?
   (a) $\nabla \cdot \mathbf{F} = 0$
   (b) $\nabla \times \mathbf{F} = 0$
   (c) $\mathbf{F} = 0$
   (d) $\|\mathbf{F}\| = 1$
   (e) $\nabla(\nabla \cdot \mathbf{F}) = 0$

6. For the series $\sum(-1)^n/n^2$, which of the following is true?

   (a) It diverges conditionally and absolutely
   (b) It converges conditionally and absolutely
   (c) It converges conditionally but diverges absolutely
   (d) It diverges conditionally but converges absolutely
   (e) It both converges and diverges

7. Evaluate $1 + \cos^2\theta + \cos^4\theta + \cos^6\theta + \cdots$, where $0 < \theta < \pi/2$.

   (a) $\sin^2\theta$
   (b) $\cos^2\theta$
   (c) $\sec^2\theta$
   (d) $\csc^2\theta$
   (e) The sum diverges

8. Given $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n^4} = \frac{\pi^4}{90}$, what is $\displaystyle\frac{1}{2^4} + \frac{1}{4^4} + \frac{1}{6^4} + \cdots$?

   (a) $\pi^4/45$
   (b) $\pi^4/90$
   (c) $\pi^4/180$
   (d) $\pi^4/1440$
   (e) The sum diverges

9. (Bonus Problem) At a convention of 100 politicians, each is either crooked or honest. Given any two politicians, at least one of them is crooked. If at least one politician is honest, how many are crooked?

   (a) 1
   (b) 49
   (c) 50
   (d) 51
   (e) 99

Free Response

1. Let S be the upper half ($z \geq 0$) of the sphere $x^2 + y^2 + z^2 = 1$, oriented upward. Evaluate

$$\iint_S \left(x, \arctan e^z, x^2 + y^2\right) \cdot d\mathbf{A}.$$

Note that S is not a closed surface.

2. Let S be a smooth oriented surface with smooth boundary ∂S. Given smooth functions f and g, show that

$$\oint_{\partial S} (f\nabla g + g\nabla f) \cdot d\mathbf{r} = 0$$

3. Determine, with proof, whether or not this series converges.

$$\sum \left(1 - \frac{1}{n}\right)^{n^2}$$

4. Recall the Fibonacci numbers: $F_0 = F_1 = 1$ and $F_{n+2} = F_{n+1} + F_n$ for all integers n. Evaluate

$$\sum_{n=0}^{\infty} \frac{F_n}{F_{n+2}F_{n+1}}.$$