Math 5C Spring 2010
Exam 2
Version B

May 7, 2010

M. Choice	
F. Resp. 1	
F. Resp. 2	
F. Resp. 3	
F. Resp. 4	
Total	

Name______________________________

Perm No.______________________________

Directions:

1. There are 125 points on this exam; 100 points = 100%.
2. Each multiple choice problem is 5 points.
3. Each multiple choice problem has exactly one best answer.
4. No multiple choice problem requires heavy computation.
5. Each free response problem is 20 points.
6. Free response questions require justification; no work, no credit.
7. **A blank free-response problem is awarded 5 points.**
8. You are allowed one (1) 3 by 5 notecard.
9. No other notes, books, or electronic devices are allowed.

Potentially useful formulas:

$$\nabla \cdot (f\mathbf{G}) = \nabla f \cdot \mathbf{G} + f\nabla \cdot \mathbf{G}$$
$$\nabla \times (f\mathbf{G}) = \nabla f \times \mathbf{G} + f\nabla \times \mathbf{G}$$

Multiple Choice

1. Split the unit sphere centered at the origin into two hemispheres, S and T, both oriented outward. If the flux of $\nabla \times \mathbf{F}$ through S is 10, what is the flux of $\nabla \times \mathbf{F}$ through T?
   (a) 0
   (b) 10
   (c) -10
   (d) Cannot be determined
   (e) None of the above

2. Given $x > 0$, evaluate $e^{-x} + e^{-2x} + e^{-3x} + \cdots$
   (a) 1
   (b) $(1 - e^{-x})^{-1}$
   (c) $(e^{-x} - 1)^{-1}$
   (d) $(1 - e^{x})^{-1}$
   (e) $(e^{x} - 1)^{-1}$

3. Let f be a smooth scalar function. Which of these has nonzero curl?
   (a) ∇f
   (b) $f\nabla f$
   (c) $\nabla \times \nabla f$
   (d) $\nabla \times (f\nabla f)$
   (e) None of the above

4. Suppose that for all positive integers N,

$$\sum_{n=0}^{N} a_n = e^{-N}\left(\frac{1}{\sqrt{N+1}} - \frac{\ln N}{N} + \frac{2^N}{N!}\right) + 1$$

   Evaluate $\sum_0^\infty a_n$.
   (a) 0
   (b) 1
   (c) $\sqrt{\pi\gamma e} + 1$
   (d) The series diverges
   (e) None of the above

5. Which of the following vector fields is path-independent?
   (a) $(z, 0, 0)$
   (b) $(z, x, 0)$
   (c) (z, y, x)
   (d) (z, x, y)
   (e) $(0, y, x)$

6. The sequences (a_n) and (b_n) satisfy $0 \le a_n \le b_n$ for all n. Which statement is true?

   (a) If $\sum a_n$ diverges, so does $\sum (-1)^n a_n$
   (b) If $\sum b_n$ converges, so does $\sum (-1)^n a_n$
   (c) If $\sum a_n$ converges, so does $\sum 1/a_n$
   (d) If $\sum b_n$ diverges, so does $\sum a_n$
   (e) None of the above

7. The region R in $\mathbb{R}^2$ satisfies

$$\oint_{\partial R} (x + 2y)\, dx + (3x + 4y + 5)\, dy = 120.$$

   What is the area of R?

   (a) 8
   (b) 24
   (c) 120
   (d) Cannot be determined
   (e) None of the above

8. A sequence of positive numbers is such that $\sqrt[n]{a_n} \to 2$ when $n \to \infty$. Which of the following series converges absolutely?

   (a) $\sum a_n$
   (b) $\sum (-1)^n a_n$
   (c) $\sum a_n/2$
   (d) $\sum \sqrt[n]{a_n}$
   (e) None of the above

9. (Bonus Problem) A sign on the wall has 100 (logically consistent) statements. They read:

   - At least one of these statements is false.
   - At least two of these statements are false.
   - At least three of these statements are false.

     ⋮
   - All of these statements are false.

   How many of the statements are true?

   (a) 0
   (b) 1
   (c) 50
   (d) 99
   (e) None of the above

Free Response

1. Let B be a ball in $\mathbb{R}^3$. Given smooth scalar functions f and g, show that

$$\iint_{\partial B} (f\nabla g - g\nabla f) \cdot d\mathbf{A} = \iiint_B (f\Delta g - g\Delta f)\, dV$$

2. Let S be the cone $z = \sqrt{x^2 + y^2}$, $0 \leq z \leq 1$, oriented upward. Define $\mathbf{G} = (-y, x, xyze^{x^2})$ and evaluate

$$\iint_S z^2(\nabla \times \mathbf{G}) \cdot d\mathbf{A}$$

Consider integration by parts.

3. Recall that

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}.$$

Evaluate the series

$$S = 1 - \frac{1}{4} + \frac{1}{9} - \frac{1}{16} + \frac{1}{25} - \frac{1}{36} + \cdots$$

4. The binomial coefficients are defined as $\binom{n}{k} = \frac{n!}{k!(n-k)!}$. Determine whether or not this series converges:

$$\sum \binom{2n}{n}^{-1}$$