Math 5C Spring 2010
Exam 3
Version A

May 28, 2010

M. Choice	
F. Resp. 1	
F. Resp. 2	
F. Resp. 3	
F. Resp. 4	
Total	

Name______________________________

Perm No.____________________________

Directions:

1. There are 125 points on this exam; 100 points = 100%.
2. Each multiple choice problem is 5 points.
3. Each multiple choice problem has exactly one best answer.
4. No multiple choice problem requires heavy computation.
5. Each free response problem is 20 points.
6. Free response questions require justification; no work, no credit.
7. **A blank free-response problem is awarded 5 points.**
8. You are allowed one (1) 3 by 5 notecard.
9. No other notes, books, or electronic devices are allowed.

Multiple Choice

1. Which of the following functions is <u>not</u> harmonic on $\mathbb{R}^2$?

   (a) x
   (b) y
   (c) xy
   (d) $x^2 - y^2$
   (e) $x^2 + y^2$

2. If 2 terms of the Taylor series are used to approximate $\cos(0.1)$, what bound does the remainder theorem gives us for the error?

   (a) 0
   (b) 1/6000
   (c) 1/6
   (d) $(.1)^3$
   (e) 1

3. The 2π-periodic function f is given by

$$f(x) = \begin{cases} 1, & 0 < x < \pi \\ -1, & -\pi < x < 0 \end{cases}$$

   At $x = 0$, to what does the Fourier series of f converge?

   (a) -1
   (b) 0
   (c) 1
   (d) 1/2
   (e) It doesn't converge

4. Using the standard inner product for functions on $[-\pi, \pi]$, what is $\|1\|$?

   (a) 0
   (b) 2π
   (c) 1
   (d) Undefined
   (e) None of the above

5. Which of the following could <u>not</u> be the interval of convergence for the series $\sum a_n x^n$?

   (a) $\{0\}$
   (b) $\mathbb{R}$
   (c) $(-1, 1]$
   (d) $(-2, 3)$
   (e) $[-4, 4]$

6. Find $f^{(7)}(0)$ for $f(x) = \sum_{n=0}^{\infty} \frac{n!}{n^n} x^n$.

   (a) $1/7^7$
   (b) $7!/7^7$
   (c) $(7!)^2/7^7$
   (d) 7^7
   (e) None of the above

7. The series $\sum a_n 3^n$ converges. Of the following, which must also converge?

   (a) $\sum a_n(-3)^n$
   (b) $\sum a_n 2^n$
   (c) $\sum a_n 4^n$
   (d) $\sum a_n^{1/3}$
   (e) $\sum e^{-a_n}$

8. The harmonic functions f and g are equal on the surface of a ball in $\mathbb{R}^3$. Which of the following must be true?

   (a) fg is harmonic
   (b) $f = g$ everywhere in the ball
   (c) The volume integral of $f + g$ over the ball is zero
   (d) Either f or g must be zero at some point
   (e) The gradients ∇f and ∇g are orthogonal

9. (Bonus Problem) Two signs are next to each other on a wall. Each has the same three statements written on it:

   - All of the other sign's statements are false.
   - Exactly two of the other sign's statements are false.
   - Exactly one of the other sign's statements are false.

   How many of the six statements are true?

   (a) 0
   (b) 2
   (c) 4
   (d) 6
   (e) It cannot be determined

Free Response

1. Evaluate the limit
$$\lim_{n\to\infty} n\left(\left(1+\frac{1}{n}\right)^n - e\right)$$

2. The Bernoulli numbers are defined by the formula

$$\frac{x}{e^x - 1} = \sum_{n=0}^{\infty} \frac{B_n}{n!} x^n$$

Use this to evaluate the series

$$\sum_{n=0}^{\infty} \frac{(n+1)B_n}{n!}$$

3. Compute the Fourier series for the 2π-periodic function f, where $f(x) = x^2$ for $-\pi < x < \pi$.

4. Let B be the unit ball in $\mathbb{R}^3$ centered at $(1, 2, 2)$ and define $f(x, y, z) = x \ln(y^2 + z^2)$. Explaining your reasoning, evaluate

$$\iint_{\partial B} f \, d\sigma.$$