Math 5C Spring 2010
Exam 3
Version B

May 28, 2010

M. Choice	
F. Resp. 1	
F. Resp. 2	
F. Resp. 3	
F. Resp. 4	
Total	

Name______________________________

Perm No.___________________________

Directions:

1. There are 125 points on this exam; 100 points = 100%.
2. Each multiple choice problem is 5 points.
3. Each multiple choice problem has exactly one best answer.
4. No multiple choice problem requires heavy computation.
5. Each free response problem is 20 points.
6. Free response questions require justification; no work, no credit.
7. **A blank free-response problem is awarded 5 points.**
8. You are allowed one (1) 3 by 5 notecard.
9. No other notes, books, or electronic devices are allowed.

Multiple Choice

1. What is the radius of convergence for the series $1 + 2x + 3x^2 + 4x^3 + 5x^4 + \cdots$?
   (a) 0
   (b) 1
   (c) n
   (d) ∞
   (e) None of the above

2. If $\sin x$ is expanded in a Fourier series, what is the coefficient of $\sin 2x$?
   (a) 0
   (b) 1
   (c) 2
   (d) 1/2
   (e) None of the above

3. For which a is $x^4 - ax^2y^2 + y^4$ harmonic in $\mathbb{R}^2$?
   (a) 0
   (b) 1
   (c) 2
   (d) 4
   (e) None of the above

4. Given $f(x) = x^5/(1-x)$, what is $f^{(9)}(0)$?
   (a) 0
   (b) 9!
   (c) 9!/4
   (d) 9!/5
   (e) None of the above

5. For which real numbers a does this series converge?

$$\sum \left[\ln\left(1 + \frac{1}{n^a}\right) - \frac{1}{n^a} \right]$$

   (a) $a > -1$
   (b) $a > 0$
   (c) $a > 1/2$
   (d) $a > 1$
   (e) None of the above

6. Let $f(x) = e^x$ for $-\pi < x < \pi$ and $g(x)$ be the Fourier series of f. What is $g(\pi)$?

   (a) e^{π}
   (b) $e^{-\pi}$
   (c) $\cosh \pi$
   (d) $\sinh \pi$
   (e) None of the above

7. If $\{v_1, v_2\}$ is an orthogonal basis in an inner product space and an arbitrary vector v is written as $v = c_1 v_1 + c_2 v_2$, what is c_1?

   (a) $\langle v_1, v_2 \rangle$
   (b) $\langle v, v_1 \rangle$
   (c) $\langle v, v_2 \rangle$
   (d) $\dfrac{v - c_2 v_2}{v_1}$
   (e) None of the above

8. Let S and T be spheres centered at the origin in $\mathbb{R}^3$ with radii 1 and 2, respectively. Assume that f is a harmonic function such that $\iint_S f \, d\sigma = 10$. What is $\iint_T f \, d\sigma$?

   (a) 10
   (b) 20
   (c) 40
   (d) Cannot be determined
   (e) None of the above

9. (Bonus Problem) There are 10 bags, each containing 10 coins. The coins are supposed to be identical, each weighing 10 grams. However, the coins in one bag are fake—they weigh only 9 grams each though they look the same as the others. You have a scale which can weigh any collection of coins and tell you the mass in grams. What is the minimum number of weighings required to identify the bag of fake coins?

   (a) 1
   (b) 2
   (c) 5
   (d) 9
   (e) None of the above

Free Response

1. Evaluate the limit

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n^2} e^{-n}$$

2. The Fibonacci numbers can be defined by the formula

$$\frac{1}{1-x-x^2} = \sum_{n=0}^{\infty} F_n x^n$$

Use this to evaluate

$$\sum_{n=0}^{\infty} \frac{nF_n}{2^{n-1}}$$

3. Let $f(x) = x$ for $0 < x < 2\pi$. Compute the Fourier series of f.

4. Let B be a ball in $\mathbb{R}^3$. Recall the maximum principle: any harmonic function in B attains its absolute max and min on the boundary. Use this to prove the following theorem.

   If f is a harmonic function and $f = 0$ on ∂B, then $f = 0$ inside B.

   Please use complete sentences.