# Math 5C Spring 2010
# Exam 3
# Version B

**May 28, 2010**

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|---|---|
| M. Choice | |
| F. Resp. 1 | |
| F. Resp. 2 | |
| F. Resp. 3 | |
| F. Resp. 4 | |
| Total | |

**Name**______________________________

**Perm No.**______________________________

Directions:

1. There are 125 points on this exam; 100 points = 100%.
2. Each multiple choice problem is 5 points.
3. Each multiple choice problem has exactly one best answer.
4. No multiple choice problem requires heavy computation.
5. Each free response problem is 20 points.
6. Free response questions require justification; no work, no credit.
7. **A blank free-response problem is awarded 5 points.**
8. You are allowed one (1) 3 by 5 notecard.
9. No other notes, books, or electronic devices are allowed.

**Multiple Choice**

1. (B)
2. (A) $\sin x$ is orthogonal to $\sin 2x$
3. (E) $a = 6$
4. (B) $f = x^5 + x^6 + x^7 + \cdots$, so the coefficient of $x^9$ is 1. The answer is $1 \cdot 9!$.
5. (C) $\ln(1 + 1/n^a) - 1/n^a \approx -1/2n^{2a}$ for large $n$. We want $2a > 1$.
6. (C) $(e^\pi + e^{-\pi})/2$
7. (E) $c_1 = \langle v, v_1\rangle / \langle v_1, v_1\rangle$
8. (C) The integral over $S$ is $4\pi f(0,0,0)$. The integral over $T$ is $16\pi f(0,0,0)$.
9. (A) Use one coin from the first bag, two from the second, three from the third, etc. If all coins were 10 grams, the total weight would be 550 grams. If the weight is, say, 547, then there are 3 fake coins on the scale—you instantly conclude that bag three has fake coins.

**Free Response**

1. Call the limit $L$. Using the logarithm gives

$$L = \exp \lim_{n\to\infty} \ln \left[ \left(1 + \frac{1}{n}\right)^{n^2} e^{-n} \right] = \exp \lim_{n\to\infty} \left( n^2 \ln\left(1 + \frac{1}{n}\right) - n \right)$$

Since $\ln(1+t) = t - t^2/2 + O(t^3)$ for small $t$,

$$L = \exp \lim_{n\to\infty} \left( n^2 \left( \frac{1}{n} - \frac{1}{2n^2} + O(n^{-3}) \right) - n \right) = \exp \lim_{n\to\infty} \left( -\frac{1}{2} + O(n^{-1}) \right) = e^{-1/2}$$

2. Differentiate:

$$\frac{1+2x}{(1-x-x^2)^2} = \sum_{n=0}^{\infty} nF_n x^{n-1}$$

Set $x = 1/2$ to get an answer of 32.

3. The Fourier coefficients are

$$a_0 = \frac{1}{2\pi} \int_0^{2\pi} x\,dx = \pi$$
$$a_n = \frac{1}{\pi} \int_0^{2\pi} x \cos nx\,dx = 0$$
$$b_n = \frac{1}{\pi} \int_0^{2\pi} x \sin nx\,dx = -\frac{2}{n}$$

The Fourier series is

$$\pi - 2\sum_{n=1}^{\infty} \frac{\sin nx}{n}$$

4. Suppose $f$ is harmonic in $B$ and $f = 0$ on $\partial B$. The max value of $f$ occurs on $\partial B$, so $f \leq 0$ everywhere. The min value of $f$ occurs on $\partial B$, so $f \geq 0$ everywhere. Therefore $f = 0$ everywhere in $B$.