Math 5C Spring 2010
Practice Exam 1

M. Choice	
F. Resp. 1	
F. Resp. 2	
F. Resp. 3	
F. Resp. 4	
Total	

Name______________________________

Perm No.____________________________

Directions:

1. There are 125 points on this exam; 100 points = 100%.
2. Each multiple choice problem is 5 points.
3. Each multiple choice problem has exactly one best answer.
4. No multiple choice problem requires heavy computation.
5. Each free response problem is 20 points.
6. Free response questions require justification; no work, no credit.
7. **A blank free-response problem is awarded 5 points.**
8. No notes, books, or electronic devices are allowed.

Multiple Choice

1. Let S be the region inside the sphere $x^2 + y^2 + z^2 = 7$, above the $x - y$ plane, and below the cone $z = \sqrt{x^2 + y^2}$. In which coordinate system can S be described most simply?
   (a) Cartesian
   (b) Cylindrical
   (c) Spherical
   (d) Prolate spheroidal
   (e) Hypebolic

2. Does the path integral of a scalar function $\int_C f\, ds$ depend on the orientation of the curve?
   (a) It never does
   (b) It always does
   (c) It sometimes does, depending on the function f
   (d) The integral isn't defined

3. Let S be the surface $x^2 + y^2 = 4$, $0 \le z \le 3$. Using the polar angle θ and the height z as parameters, which of these gives the area element $d\sigma$?
   (a) $dz\, d\theta$
   (b) $r\, dz\, d\theta$
   (c) $2\, dz\, d\theta$
   (d) $4\, dz\, d\theta$
   (e) None of the above

4. Let S be the sphere of radius R centered at the origin. What is $\iint_S d\sigma$?
   (a) 0
   (b) $4\pi R^2$
   (c) $16\pi^2 R^4$
   (d) $4\pi R^4$
   (e) $16\pi^2 R^2$

5. The plane $z = 0$ and the surface $z = 16 - x^2 - y^2$ bound a region R in $\mathbb{R}^3$. Which of the following functions $f(x, y, z)$ gives the volume of R when we compute $\iiint_R f(x, y, z)\, dV$?
   (a) 1
   (b) V
   (c) $16 - x^2 - y^2$
   (d) $r\, dr d\theta$
   (e) The triple integral can't be used

6. Let S be the unit sphere centered at $(0, 0, 1)$. In spherical coordinates, which of the following equations describes S?

   (a) $\rho = 1$
   (b) $\rho = \cos\phi$
   (c) $\rho = 2\cos\phi$
   (d) $\rho = \sin\phi$
   (e) $\rho = 2\sin\phi$

7. Suppose that $\mathbf{F}$ is a constant vector field on $\mathbb{R}^3$ with $\|\mathbf{F}\| = 2$. Let S be the sphere of radius R centered at the origin, outwardly oriented. What is $\iint_S \mathbf{F} \cdot d\mathbf{A}$?

   (a) 0
   (b) 2
   (c) $2R$
   (d) $4\pi R^2$
   (e) $8\pi R^2$

8. If we rewrite
$$\int_0^1 \int_0^x \int_0^y f(x, y, z)\, dz\, dy\, dx$$
using the order $dx\, dy\, dz$, what is the upper limit on x?

   (a) 1
   (b) y
   (c) z
   (d) $1 - y$
   (e) $1 - z$

9. Let R be the hemisphere given by $x^2 + y^2 + z^2 \leq 4$ and $z \geq 0$. Evaluate
$$\iiint_R (x + y + xy + y^3 + 5)\, dV.$$

   (a) 0
   (b) 5
   (c) π
   (d) $2\pi/3$
   (e) None of the above

Free Response

1. Let R be the top half ($y \geq 0$) of the unit disk in $\mathbb{R}^2$, centered at the origin. Evaluate

$$\iint_R e^{-(x^2+y^2)}\, dA.$$

2. Evaluate

$$\iiint_{\mathbb{R}^3} \frac{dV}{[1+(x^2+y^2+z^2)^{3/2}]^{3/2}}.$$

3. Let S be the upper half ($z \geq 0$) of the sphere given by $x^2 + y^2 + z^2 = 1$. Evaluate $\iint_S z \, d\sigma$.

4. A curve C is parametrized by $\mathbf{r}(t) = (t^2, t, 3)$, with $0 \leq t \leq 1$. Evaluate $\displaystyle\int_C y\, ds$.