Math 5C Spring 2010
Practice Exam 1 Solutions

M. Choice	
F. Resp. 1	
F. Resp. 2	
F. Resp. 3	
F. Resp. 4	
Total	

Name________________________________

Perm No.________________________________

Directions:

1. There are 125 points on this exam; 100 points = 100%.
2. Each multiple choice problem is 5 points.
3. Each multiple choice problem has exactly one best answer.
4. No multiple choice problem requires heavy computation.
5. Each free response problem is 20 points.
6. Free response questions require justification; no work, no credit.
7. **A blank free-response problem is awarded 5 points.**
8. No notes, books, or electronic devices are allowed.

Multiple Choice

1. (C) In spherical, the region is merely $0 \leq \rho \leq \sqrt{7}$, $0 \leq \theta \leq 2\pi$, and $\pi/4 \leq \phi \leq \pi/2$.
2. (A)
3. (C) Use geometry to get the answer quickly.
4. (B) The surface area of a sphere
5. (A) No matter the region, it's the triple integral of 1 that gives volume.
6. (C) From $x^2 + y^2 + (z-1)^2 = 1$ we get $x^2 + y^2 + z^2 = 2z$. Set $x^2 + y^2 + z^2 = \rho^2$ and $z = \rho \cos \theta$.
7. (A) The $\mathbf{F}\cdot$ can come out of the integral, which is then zero by symmetry (vectors at diametrically opposite points cancel when they add).
8. (A) $0 \leq z \leq 1$, $z \leq y \leq 1$, $y \leq x \leq 1$
9. (E) By symmetry, the integrals of x, y, xy, y^3 are all zero on R. The integral of 5 is 5 times the volume of R, so the integral is $80\pi/3$.

Free Response

1. In polar coordinates,

$$\iint\limits_R e^{-(x^2+y^2)}\, dA = \int_0^\pi \int_0^1 re^{-r^2}\, dr\, d\theta = \int_0^\pi \frac{1-1/e}{2}\, d\theta = \frac{\pi}{2}\left(1-\frac{1}{e}\right).$$

2. In spherical coordinates,

$$\begin{aligned}\iiint\limits_{\mathbb{R}^3} \frac{dV}{[1+(x^2+y^2+z^2)^{3/2}]^{3/2}} &= \int_0^{2\pi}\int_0^\pi\int_0^\infty \frac{\rho^2\sin\phi}{(1+\rho^3)^{3/2}}\, d\rho\, d\phi\, d\theta \\ &= \int_0^{2\pi}\int_0^\pi \frac{2}{3}\sin\phi\, d\phi\, d\theta \\ &= \frac{8\pi}{3}\end{aligned}$$

3. Use the standard sphere parametrization: $\mathbf{r}(\theta,\phi) = (\cos\theta\sin\phi, \sin\theta\sin\phi, \cos\phi)$ with $0 \le \theta \le 2\pi$ and $0 \le \phi \le \pi/2$. Then $d\sigma = \sin\phi\, d\phi\, d\theta$ and

$$\iint_S z\, d\sigma = \int_0^{2\pi}\int_0^{\pi/2} \cos\phi\sin\phi\, d\phi\, d\theta = \frac{1}{2}\int_0^{2\pi}\int_0^{\pi/2} \sin 2\phi\, d\phi\, d\theta = \pi.$$

4. From $\mathbf{r}'(t) = (2t, 1, 0)$ we get $ds = \|\mathbf{r}'(t)\|\, dt = \sqrt{1+4t^2}\, dt$. Since $y = t$, we get

$$\int_C y\, ds = \int_0^1 t\sqrt{1+4t^2}\, dt = (1/12)(1+4t^2)^{3/2}\Big|_0^1 = \frac{1}{12}\left(5\sqrt{5}-1\right).$$