Math 5C Spring 2010
Practice Exam 2

M. Choice	
F. Resp. 1	
F. Resp. 2	
F. Resp. 3	
F. Resp. 4	
Total	

Name______________________________

Perm No.______________________________

Directions:

1. There are 125 points on this exam; 100 points = 100%.
2. Each multiple choice problem is 5 points.
3. Each multiple choice problem has exactly one best answer.
4. No multiple choice problem requires heavy computation.
5. Each free response problem is 20 points.
6. Free response questions require justification; no work, no credit.
7. **A blank free-response problem is awarded 5 points.**
8. You are allowed one (1) 3 by 5 notecard.
9. No other notes, books, or electronic devices are allowed.

Multiple Choice

1. A series $\sum a_n$ of positive terms converges. Which of the following must <u>not</u> converge?
   (a) $\sum a_n^2$
   (b) $\sum |a_n|$
   (c) $\sum (-1)^n a_n$
   (d) $\sum 2a_n$
   (e) $\sum 1/a_n$

2. Which of the following is always zero?
   (a) $\nabla \times (\nabla \times \mathbf{F})$
   (b) $\nabla \cdot (\nabla f)$
   (c) $\nabla \times (\nabla f)$
   (d) $\nabla(\nabla \cdot \mathbf{F})$
   (e) $\Delta^2 f$

3. Suppose that $\sum b_n$ is a convergent series of positive terms. Given another positive series $\sum a_n$, which of the following implies its convergence?
   (a) $a_n \to 1$
   (b) $a_{n+1}/a_n \to 1$
   (c) $\sqrt[n]{a_n} \to 1$
   (d) $b_n/a_n \to 1$
   (e) $\displaystyle\int_1^\infty a_n \, dn < \infty$

4. Let R be a region in $\mathbb{R}^2$ with piecewise smooth boundary ∂R, positively oriented. Find the area of R, given that

$$\oint_{\partial R} 2y \, dx + 5x \, dy = 9.$$

   (a) 0
   (b) 3
   (c) 6
   (d) 9
   (e) Cannot be determined without more information

5. A geometric series consisting of positive numbers is such that the sum of the first two terms is half the sum of the entrie series. If the first term is 1, what is the second term?
   (a) 2
   (b) $\sqrt{2}$
   (c) $1/\sqrt{2}$
   (d) $1/2$
   (e) None of the above

6. Which of the following vector fields is *not* path-independent?

   (a) $(1,0,0)$
   (b) $(1,1,z)$
   (c) (x,y,z)
   (d) (z,x,y)
   (e) $(\arctan(x), e^y, \ln(1+z^2))$

7. Which of the following series converges conditionally but not absolutely?

   (a) $\sum(-1)^n n^{-2}$
   (b) $\sum(-1)^n(1-1/n)^n$
   (c) $\sum(-1)^n 2^{-n}$
   (d) $\sum(-1)^n \ln n$
   (e) $\sum(-1)^n \sqrt{n}(1+n)^{-1}$

8. A smooth path-independent vector field defined on all of $\mathbb{R}^3$ is a

   (a) gradient
   (b) curl
   (c) divergence
   (d) constant
   (e) flux

9. Five pirates—named A, B, C, D and E—want to split 100 gold coins amongst themselves. They proceed as follows. Pirate A suggests a distribution of coins and all five pirates vote on whether they like the proposal. If a strict majority likes the proposal, the money is distributed and everyone leaves. Otherwise pirate A is thrown overboard (with no money) and pirate B steps up to propose a coin allocation. The process continues the same way, etc. Assume the pirates are perfectly logical, they want as much money as possible, and that when all else is equal they'll vote to throw someone overboard. In his proposal, how much money should pirate A allocate to himself to maximize his share?

   (a) 98
   (b) 97
   (c) 50
   (d) 1
   (e) None of the above

Free Response

1. Let R be the unit ball in $\mathbb{R}^3$ centered at the origin. Define the functions

$$f(x, y, z) = 1 - x^2 - y^2 - z^2 \qquad \text{and} \qquad \mathbf{G} = (e^{y^2}, \arctan(z^3), z)$$

Use integration by parts to evaluate

$$\iiint_R \nabla f \cdot \mathbf{G} \, dV.$$

2. The main loop in the Folium of Descartes is given by $x = 3t/(1 + t^3)$, $y = 3t^2/(1 + t^3)$, $0 < t < \infty$. Find the enclosed area.

3. Determine (with justification) whether or not this series converges.

$$\sum \frac{1}{n} \ln\left(1 + \frac{1}{n}\right)$$

4. Let $|r| < 1$. Evaluate $\displaystyle\sum_{n=0}^{\infty} nr^n$.