Math 5C Spring 2010
Practice Exam 3

M. Choice	
F. Resp. 1	
F. Resp. 2	
F. Resp. 3	
F. Resp. 4	
Total	

Name______________________________

Perm No.___________________________

Directions:

1. There are 125 points on this exam; 100 points = 100%.
2. Each multiple choice problem is 5 points.
3. Each multiple choice problem has exactly one best answer.
4. No multiple choice problem requires heavy computation.
5. Each free response problem is 20 points.
6. Free response questions require justification; no work, no credit.
7. **A blank free-response problem is awarded 5 points.**
8. No notes, books, or electronic devices are allowed.

Multiple Choice

1. Using the standard inner product for functions on $[-\pi, \pi]$, what is $\|\sin x\|$?
   (a) 0
   (b) $\sin^2 x$
   (c) 1
   (d) Undefined
   (e) None of the above

2. The function $f(x) = x^2$, $-\pi < x < \pi$ is expanded into a Fourier series. What is the coefficient of $\sin(3x)$?
   (a) 0
   (b) 3
   (c) 1/3
   (d) 9
   (e) 1/9

3. The power series $\sum a_n x^n$ has a finite radius of convergence $R > 0$. What is the radius of convergence of $\sum a_n (x/2)^n$?
   (a) R
   (b) $2R$
   (c) $R/2$
   (d) ∞
   (e) Cannot be determined without more information

4. Suppose that f is harmonic on $\mathbb{R}^3$ and that $f = 6$ for all points on the unit sphere centered at the origin. What is the largest possible value of $f(1/2, 0, 0)$?
   (a) 0
   (b) 6
   (c) 84π
   (d) It could be arbitrarily large
   (e) None of the above

5. By definition, a harmonic function f is one which satisfies which of the following?
   (a) $\nabla f = 0$
   (b) $\Delta f = 0$
   (c) $\nabla \times \nabla f = 0$
   (d) $\Delta^2 f = 0$
   (e) $df = 0$

6. Which of these is a series expansion of $-\ln(1-x)$ for x near zero?
   (a) $x + x^2/2 + x^3/3 + \cdots$
   (b) $x - x^2/2 + x^3/3 - \cdots$
   (c) $-x + x^2/2 - x^3/3 + \cdots$
   (d) $-x - x^2/2 - x^3/3 - \cdots$
   (e) None of the above
7. When $1/(1-x)$ is expanded in a Taylor series at zero, what is its interval of convergence?
   (a) $(-1, 1)$
   (b) $(-1, 1]$
   (c) $[-1, 1)$
   (d) $[-1, 1]$
   (e) $\mathbb{R}$
8. The Riemann-Lebesgue lemma states that, if $\int_{-\pi}^{\pi} |f(x)|\, dx < \infty$, then

   $$\lim_{n\to\infty} \int_{-\pi}^{\pi} f(x) \cos nx\, dx = \lim_{n\to\infty} \int_{-\pi}^{\pi} f(x) \sin nx\, dx = 0.$$

   What does this tell about the Fourier series of f?
   (a) Nothing
   (b) Sums and integrals can be interchanged
   (c) The series converges
   (d) Parseval's identity
   (e) The coefficients tend to zero
9. The 2π-periodic function $f(x) = x^3$, $-\pi < x < \pi$, is expanded in a Fourier series. What is the value of the series at $x = \pi$?
   (a) 0
   (b) π^3
   (c) $-\pi^3$
   (d) The series diverges
   (e) None of the above

Free Response

1. Evaluate
$$\lim_{n\to\infty} n^2 \left[\left(1+\frac{1}{n}\right)^{n+1/2} - e \right]$$

2. The generating function for the Catalan numbers is

$$\sum_{n=0}^{\infty} \frac{1}{n+1}\binom{2n}{n} x^n = \frac{1-\sqrt{1-4x}}{2x}$$

Use this to sum the series

$$\sum_{n=0}^{\infty} \binom{2n}{n} 2^{-4n}$$

3. Given

$$\pi - |x| = \frac{\pi}{2} + \frac{4}{\pi}\left(\cos x + \frac{\cos 3x}{9} + \frac{\cos 5x}{25} + \frac{\cos 7x}{49} + \cdots\right), \qquad -\pi < x < \pi,$$

use Parseval's identity to evaluate

$$1 + \frac{1}{3^4} + \frac{1}{5^4} + \frac{1}{7^4} + \cdots$$

4. Let f be a harmonic function in $\mathbb{R}^3$ and B be the unit ball centered at the origin. Assume that (in spherical coordinates) $f = \sin\phi$ on ∂B. Find $f(0,0,0)$.