Math 8, Summer 2012
Practice Exam 1

Short Ans.	
1	
2	
3	
Total	

Name————————————————————

Perm No.————————————————————

Directions:

1. Each problem is graded out of 4 points.

2. Each short answer question is worth 1 point.

3. You're only allowed a writing instrument and your wits.

4. Proofs should be clean, to the point, and written in proper English sentences.

Short Answer

1. (A logic puzzle) Among the following three statements, indicate which one(s) is/are true.

   - All three of these statements are false.
   - Exactly two of these statements are false.
   - Exactly one of these statements is false.

2. Let A be a set. Give a precise definition of $\mathcal{P}(A)$.

3. Give a precise definition of what it means for $f : A \to B$ to be bijective.

4. Let P and Q be statements. Under what circumstances is $(P$ or $Q)$ false?

5. Give a precise definition of $\varnothing$.

6. Give an example of a function $f : \mathbb{R} \to \mathbb{R}$ which is surjective but not injective.

7. Using the axiom of choice we can reorder $\mathbb{R}$ so that every subset of $\mathbb{R}$ has a least element. We can define a function that produces the least element from a given subset. Specify the domain and codomain of this function.

8. Give a precise definition of what it means for an integer n to be even.

Problems

1. Prove that $\log_2(3)$ is irrational.

2. Prove that

$$\bigcup_{n=1}^{\infty} \bigcap_{m=n}^{\infty} \{1, 2, 3, \ldots, m\} = \mathbb{N}$$

3. Let $f : A \to B$ be a function. Prove that f is injective if and only if for any sets $S, T \subseteq A$ we have $f(S \cap T) = f(S) \cap f(T)$.