MATH 118A, FALL 2014, PROBLEM SET 1
DUE WEDNESDAY, OCTOBER 8

Note: This problem set is worth only half as much as a typical problem set.

1. *Fun with rationals*

(a) [Rudin 1.1] If r is rational and nonzero and x is irrational, prove that $r + x$ and rx are irrational.

(b) Prove that there is no rational number whose cube is 2.

Challenge (not for credit): Prove that the set of rational numbers whose cube is less than 2 has no largest element.

2. *A warning about swapping orders of limits*

Compute the following two integrals. Your answers should be explicit numbers. Simply compute as you would in Math 6A. Show your work, but you need not formally prove anything.

Be sure not to change variables or swap the order of integration: compute the inner integral first, then the outer integral.

(a)

$$\int_1^\infty \left(\int_1^\infty \frac{x^2 - y^2}{(x^2 + y^2)^2}\, dy \right) dx$$

(b)

$$\int_1^\infty \left(\int_1^\infty \frac{x^2 - y^2}{(x^2 + y^2)^2}\, dx \right) dy$$

Hint: Compute $\frac{\partial}{\partial y} \left(\frac{y}{x^2+y^2} \right)$ to help with (a), and something similar to help with (b). If you don't recall how to integrate the single integrals you get, feel free to look at a standard integral table.