

How to discover outer space. And, while you're there, relative train track maps.

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Culler and Vogtmann introduced the outer space of a free group F_n for the immediate purpose of computing the virtual cohomological dimension of $Out(F_n)$. A little later, Bestvina and Handel introduced relative train track maps for the immediate purpose of solving the Scott Conjecture for elements of $Aut(F_n)$. We shall present an “alternate history”. In the first hour we show how outer space and Stallings fold paths could have been discovered by studying classical problems of Nielsen and Whitehead. In the second hour, we show how relative train track maps could have been discovered by studying growth properties of outer automorphisms. In the third hour, we show how the latest generation of relative train track maps — the “CTs” — could have been discovered by counting periodic orbits of relative train track maps.