## 1. Hand in Problem from Set 3

We provide two solutions. In Solution 1 we choose to label the distance from the top of the roof to the falling ball by $x$, where $x > 0$ corresponds to being below the roofline. In Solution 2 we choose to work with the height of the ball above the ground instead.

### 1.1. Solution 1.

We ask how long it takes a ball which is dropped from the roof of a 100ft tall building to hit the ground. To model this, let $x$ denote the distance from the top of the building to the falling ball measured in feet. Thus $x = 0$ at the instant the ball is turned loose and $x = 100$ corresponds to the ball hitting the ground. As acceleration due to gravity is 32.2 ft/s$^2$, we have

$$\frac{d^2x}{dt^2} = 32.3, \tag{1}$$

where "$t$" is the elapsed time measured in seconds after the ball is turned loose. This equation follows from the very meaning of acceleration: the rate of change of velocity with respect to time. Of course, the velocity of the ball is $v = dx/dt$.

We use (1) to determine when the ball hits the ground. Integrating it once we find

$$\frac{dx}{dt} = 32.2t + \frac{dx}{dt}(0).$$

As we are assuming that the ball is "dropped" and not thrown, it's velocity at time 0 is 0. Using this, we integrate again to find

$$x(t) = 16.1t^2 + x(0) = 16.1t^2$$

because $x(0) = 0$. The ball hits the ground when $x(t) = 100$ or $100 = 16.2t^2$. Solving for $t$ we find $t = \sqrt{1000/16.1} = 2.492223931....$

If a ball is dropped and the time it takes to hit the ground is significantly different than this result, it must be that (1) is not accurate. This means that there are other forces acting on the ball besides gravity. The most significant force we have neglected is friction - after all, we know this force is very considerable at high speeds.

### 1.2. Solution 2.

We ask how long it takes a ball which is dropped from the roof of a 100ft tall building to hit the ground. To model this, let $h$ denote the distance from the ground to the falling ball measured in feet. Thus $h = 100$ at the instant the ball is turned loose and $h = 0$ corresponds to the ball hitting the ground. As acceleration due to gravity is 32.2 ft/s$^2$ and acts to accelerate the ball *toward the ground* (decreasing $h$), we have

$$\frac{d^2h}{dt^2} = -32.3, \tag{2}$$

where "$t$" is the elapsed time measured in seconds after the ball is turned loose. This equation follows from the very meaning of acceleration: the rate of change of velocity with respect to time. Of course, the velocity of the ball is $v = dh/dt$.

We use (2) to determine when the ball hits the ground. Integrating it once we find

$$\frac{dh}{dt} = -32.2t + \frac{dh}{dt}(0).$$

As we are assuming that the ball is "dropped" and not thrown, it's velocity at time 0 is 0. Using this, we integrate again to find

$$h(t) = -16.1t^2 + h(0) = -16.1t^2 + 100$$

because $h(0) = 100$. The ball hits the ground when $h(t) = 0$ or $100 = 16.2t^2$. Solving for $t$ we find $t = \sqrt{1000/16.1} = 2.492223931....$

If a ball is dropped and the time it takes to hit the ground is significantly different than this result, it must be that (1) is not accurate. This means that there are other forces acting on the ball besides gravity. The most significant force we have neglected is friction - after all, we know this force is very considerable at high speeds.