1. do Carmo, p46, 7.

2. do Carmo, p56, 2 (assume that the connection is affine, instead of Riemannian).

3. do Carmo, p57, 3 (again, deal with affine connections only).

4. Let \((M^n, g)\) be a Riemannian manifold and \(f\) a smooth function on \(M\). Define the gradient \(\nabla f\) of \(f\) to be the vector field such that

\[
\langle \nabla f, X \rangle = X f \equiv df(X),
\]

for any vector field \(X\) on \(M\). Let \(x = (x_1, \cdots, x_n)\) be a local coordinate system on \(M\) and \(g = g_{ij} dx_i dx_j\). Define \(g^{ij}\) to be the entries of the inverse of the matrix \((g_{ij})\):

\[
(g^{ij}) = (g_{ij})^{-1}.
\]

Show that

\[
\nabla f = (g^{ij} \frac{\partial f}{\partial x_i}) \frac{\partial}{\partial x_j}.
\]

Hence it agrees with the usual gradient in the Euclidean space.