Math 4A – Xianzhe Dai

UCSB

April 16, 2014

Based on the 2013 Millett and Scharlemann Lectures

Definition

A linear transformation is a function $T : \mathbb{R}^n \to \mathbb{R}^m$ with these properties:

- For any vectors $\vec{u}, \vec{v} \in \mathbb{R}^n$, $T(\vec{u} + \vec{v}) = T(\vec{u}) + T(\vec{v})$
- For any vector $\vec{u} \in \mathbb{R}^n$ and any $c \in \mathbb{R}$, $T(c\vec{u}) = cT(\vec{u})$.

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By using standard basis, all linear transformations are matrix transformation

$$\vec{e}_1 = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \ \vec{e}_2 = \begin{bmatrix} 0 \\ 1 \\ \vdots \\ 0 \end{bmatrix}, \dots, \vec{e}_n = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}$$

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$$\vec{x} = x_1 \vec{e}_1 + x_2 \vec{e}_2 + \ldots + x_n \vec{e}_n$$

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Suppose $T : \mathbb{R}^n \to \mathbb{R}^m$ is a (matrix) linear transformation.

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This means that the reduced echelon form of the matrix of T must have a pivot in each of its n columns (no free variables), i.e. = 900 exactly *n* non-zero rows.

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Examples: shears, contractions and expansions, rotations,

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$$\{a_{11}, a_{22}, ..., a_{mm}\}$$

are the diagonal entries (if $m \le n$; if n < m must stop at a_{nn})

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Also: upper triangular and lower triangular:

$$\begin{bmatrix} a_{11} & * & \cdots & * \\ 0 & a_{22} & \cdots & * \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & a_{mm} \end{bmatrix} \qquad \begin{bmatrix} a_{11} & 0 & \cdots & 0 \\ * & a_{22} & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ * & * & \cdots & a_{mm} \end{bmatrix}$$

Matrix addition: Can add two matrices of the same size! entry by entry.

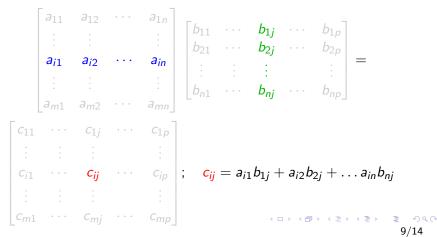
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Another interpretation: suppose A is an $m \times n$ matrix and

$$B = \begin{bmatrix} \vec{b}_1 & \vec{b}_2 & \cdots & \vec{b}_n \end{bmatrix}$$

is an $n \times p$ matrix, then AB is the $m \times p$ matrix given by:

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Major warning: Matrix multiplication is not commutative. That is, in general, $AB \neq BA$

Other strange properties:

- Can't cancel: If AB = AC it does not follow that B = C, even when $A \neq 0$
- Can multiply to zero: If AB = 0 it does not follow that A = 0 or B = 0

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Example of both: Let

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}; \quad B = \begin{bmatrix} 0 & 12 \\ 0 & 0 \end{bmatrix}; \quad C = \begin{bmatrix} 0 & 5 \\ 0 & 0 \end{bmatrix}.$$

$$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 12 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 5 \\ 0 & 0 \end{bmatrix}$$

But mostly the properties are nice:

TheoremIf A, B, and C are sized so that the products are defined, and $t \in \mathbb{R}$, then**1** A(BC) = (AB)C (associative),**2** A(B+C) = AB + AC (distributes),**3** (B+C)A = BA + CA (distributes),**4** (AB) = (tA)B = A(tB) (associative)

Only the first rule is not easy to see (it takes a bit of computation).

Definition

The $n \times n$ diagonal matrix

$$I_n = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & 1 \end{bmatrix}$$

is called the identity matrix.

Easy to check:

- For A any $m \times n$ matrix, $AI_n = A$.
- For B any $n \times p$ matrix, $I_n B = B$.
- So if A is a square $n \times n$ matrix then $I_n A = A = A I_n$.

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The last matrix operation is the matrix transpose.

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It enjoys the following properties.

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$$(A + B)^{T} = A^{T} + B^{T},$$

$$(sA)^{T} = sA^{T}.$$

$$(AB)^{T} = B^{T}A^{T}.$$
 (Order of multiplication reversed!)

Check by computations