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Last time:

- Harmonic Oscillator

$$\boxed{m\ddot{x} + b\dot{x} + kx = f(t)}$$

mass

damping const.

spring const.

forcing term

— 2^{nd} order linear diff. eq$^{\underline{n}}$ (w. const. coeff's)

- Solving unforced harmonic oscillation

$$m\ddot{x} + b\dot{x} + kx = 0$$

Educated guess: try $x = e^{rt}$ w. r to be determined (— method of undetermined coeff's)

Plug into eq$^{\underline{n}}$ $\Longrightarrow$ quadratic eq$^{\underline{n}}$ for r:

$$\boxed{mr^2 + br + k = 0}$$

characteristic eq$^{\underline{n}}$

$\Longrightarrow$ r_1, r_2 roots.

$$m\ddot{x} + b\dot{x} + kx = 0$$ 1/11/12

General sol^{n}s:

1). characteristic eq^{n}

$$mr^2 + br + k = 0$$

$\Rightarrow$ two roots r_1, r_2.

2) two distinct real roots r_1, r_2, $r_1 \neq r_2$

general sol^{n}

$$x(t) = c_1 e^{r_1 t} + c_2 e^{r_2 t}$$

3). two identical (real) roots $r_1 = r_2$

general sol^{n}:

$$x(t) = c_1 e^{r_1 t} + c_2 t e^{r_1 t}$$

fudge factor

4). complex roots —

to be continued!

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Sol$^{\underline{n}}$s for linear homogeneous diff. eq$^{\underline{n}}$s.

linear $\left\{ \boxed{y'' + p(t)\, y' + q(t)\, y = 0} \right.$ ← homogeneous

need not be const. !

—— Uniqueness and Existence Theorem: assuming $p(t)$, $q(t)$ continuous on (a,b) and $t_0 \in (a,b)$. then there is a unique sol$^{\underline{n}}$ of the diff. eq$^{\underline{n}}$ satisfying the initial cond$^{\underline{n}}$s:

$$y(t_0) = A, \qquad y'(t_0) = B$$

for any $A, B \in \mathbb{R}$.

$\Longleftrightarrow$

—— Sol$^{\underline{n}}$. Space Theorem: the space of sol$^{\underline{n}}$s of the diff. eq$^{\underline{n}}$. is a vector space of dimension 2. i.e. the general sol$^{\underline{n}}$ can be written as

$$y(t) = C_1\, y_1(t) + C_2\, y_2(t)$$

for two linearly independent sol$^{\underline{n}}$s $y_1(t)$, $y_2(t)$.

Electric Circuit

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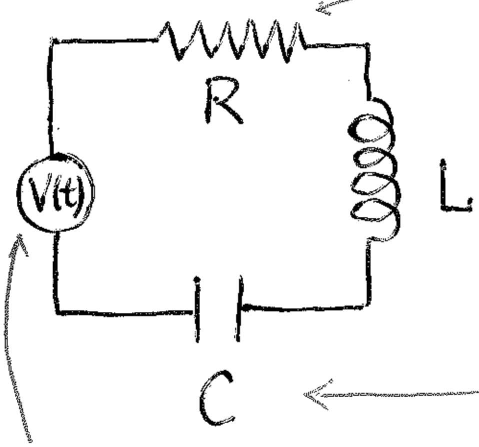

Resistor : $V = RI$ ← current (Ohm's law)

resistance (const.)

inductor : $V = L\dfrac{dI}{dt}$ (Faraday's law)

inductance (const.)

capacitor : $V = \dfrac{1}{C} Q$ ← charge

capacitance (const.)

applied voltage

— $I = \dfrac{dQ}{dt}$

— Kirchoff's law : input voltage = sum of voltage drop around circuit

$\Longrightarrow$

$$RI + L\frac{dI}{dt} + \frac{1}{C}\int I\,dt = V(t)$$

or, in terms of Q :

$$\boxed{L\ddot{Q} + R\dot{Q} + \frac{1}{C}Q = V(t)}$$

Compare w/. mass-spring system :

$$\boxed{m\ddot{x} + b\dot{x} + kx = f(t)}$$