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2^{nd} order linear homog. diff. eq$^{\underline{n}}$.
w/. const. coeff's:

$$\boxed{ay'' + by' + cy = 0}$$

a, b, c real numbers.

Char. eq$^{\underline{n}}$.

$$ar^2 + br + c = 0.$$

3. complex roots: $r_1 = \alpha + i\beta$, $r_2 = \alpha - i\beta$ (This happens when $b^2 - 4ac < 0$)

$e^{r_1 t} = e^{(\alpha + i\beta)t} = e^{\alpha t} \cdot e^{i\beta t}$

$\Rightarrow$ general sol$^{\underline{n}}$

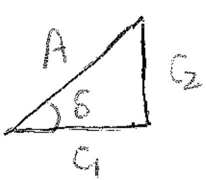
$$y(t) = e^{\alpha t}(C_1 \cos\beta t + C_2 \sin\beta t)$$

underdamped $\longrightarrow$ oscillation!

4. Phase-Amplitude form

$$y(t) = e^{\alpha t}(C_1 \cos\beta t + C_2 \sin\beta t)$$

$$= e^{\alpha t}[A\cos(\beta t - \delta)]$$



$$A = \sqrt{C_1^2 + C_2^2}, \quad \tan\delta = C_2 / C_1.$$

$$= \underbrace{Ae^{\alpha t}}_{\text{time-varying amplitude}} \cos\Big[\underbrace{\beta}_{\text{quasi-frequency}}\big(t - \underbrace{\tfrac{\delta}{\beta}}_{\text{phase shift}}\big)\Big]$$

Euler's Formula De-mystified:

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$$\boxed{e^{i\theta} = \cos\theta + i\sin\theta} \quad !?$$

Taylor expansion

$$e^{i\theta} = 1 + i\theta + \frac{1}{2!}(i\theta)^2 + \frac{1}{3!}(i\theta)^3 + \cdots$$

$$= 1 + i\theta + \frac{1}{2!}i^2\theta^2 + \frac{1}{3!}i^3\theta^3 + \cdots$$

$i^2 = -1$
$i^3 = i^2 \cdot i$
$= -i$

$$= \underbrace{1}_{\text{no } i\text{'s}} + i\theta \underbrace{- \frac{1}{2!}\theta^2}_{\text{no } i\text{'s}} - \frac{1}{3!} i\theta^3 + \underbrace{\cdots}_{\text{no } i\text{'s}}$$

$$= \underbrace{\left(1 - \frac{1}{2!}\theta^2 + \frac{1}{4!}\theta^4 - \cdots\right)}_{= \cos\theta} + i\underbrace{\left(\theta - \frac{1}{3!}\theta^3 + \frac{1}{5!}\theta^5 - \cdots\right)}_{= \sin\theta}$$

$$= \cos\theta + i\sin\theta \quad !!$$

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2^{nd} order linear nonhomog. diff. eq^{n}:

$$\boxed{y'' + p(t)\,y' + q(t)\,y = f(t)}$$

linear — nonhomog.

A general sol^{n} $y(t)$ consists of two parts:

$$\boxed{y(t) = \underbrace{y_h(t)}_{\text{homog. part}} + \underbrace{y_p(t)}_{\text{particular sol}^{n}}}$$

— $y_h(t)$ is the general sol^{n}. of the homog. eq^{n}:

$$\underline{y'' + p(t)\,y' + q(t)\,y = \underline{0}}.$$

— $y_p(t)$ is one sol^{n} of the nonhomog. eq^{n}:

$$\underline{y'' + p(t)\,y' + q(t)\,y = \underline{f(t)}}.$$

— When $p(t)$, $q(t)$ are const., we can use our previous method (characteristic eq^{n}) to solve $y_h(t)$.

— $y_p(t)$? Method of undetermined coeff's.

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Method of Undetermined Coeff's for $y_p(t)$ from the look of the nonhomog. term:

$$\boxed{ay'' + by' + cy = f(t)}$$

1°. If $f(t) = a_n t^n + a_{n-1} t^{n-1} + \cdots + a_1 t + a_0$ is a polynomial, then try

to be determined

$$y_p(t) = \underset{\sim}{A_n} t^n + \cdots + \underset{\sim}{A_1} t + \underset{\sim}{A_0}$$

2°. If $f(t) = a e^{kt}$ is an exponential f^n,

same

then try

$$y_p(t) = A e^{kt}$$

to be determined.

3°. If $f(t) = a \cos \underset{\sim}{\omega} t + b \sin \underset{\sim}{\omega} t$ is a combination of sine & cosine f^n, then try

same!

$$y_p(t) = A \cos \underset{\sim}{\omega} t + B \sin \underset{\sim}{\omega} t$$

to be determined

4°. If $f(t) =$ product, then try $y_p(t) =$ product:

e.g. $f(t) = (a_n t^n + \cdots + a_1 t + a_0) e^{kt} \implies$

$$y_p(t) = (A_n t^n + \cdots + A_1 t + A_0) e^{kt}.$$