Method of undetermined coeff's for $y_p(t)$ 1/20/12

— refined version:

$$a y'' + b y' + c y = f(t)$$

Step 1: Solve char. eq^{n}

$$ar^2 + br + c = 0 \implies r_1, r_2 \text{ two roots}$$

(we need this for y_h anyway.)

Step 2: Examine $f(t)$ to determine the form of $y_p(t)$:

— If $f(t) = (a_n t^n + \cdots + a_1 t + a_0) e^{kt}$

($k=0$ if no exp. term)

then

$$y_p(t) = \begin{cases} (A_n t^n + \cdots + A_1 t + A_0) e^{kt} & \text{if } k \neq r_1, \neq r_2 \\ t(A_n t^n + \cdots + A_1 t + A_0) e^{kt} & \text{if } k = \text{one but not both of } r_1, r_2 \\ t^2 (A_n t^n + \cdots + A_1 t + A_0) e^{kt} & \text{if } k = r_1 = r_2 \end{cases}$$

— If $f(t) = (a_n t^n + \cdots + a_1 t + a_0) e^{kt} \cos \omega t + (b_n t^n + \cdots + b_1 t + b_0) e^{kt} \sin \omega t$ $(\omega \neq 0)$

then

$$y_p(t) = \begin{cases} (A_n t^n + \cdots + A_1 t + A_0) e^{kt} \cos \omega t + (B_n t^n + \cdots + B_1 t + B_0) e^{kt} \sin \omega t & \text{if } k + i\omega \neq r_1, r_2 \\ t\left[(A_n t^n + \cdots + A_1 t + A_0) e^{kt} \cos \omega t + (B_n t^n + \cdots + B_1 t + B_0) e^{kt} \sin \omega t\right] & \text{if } k + i\omega = \text{one of } r_1, r_2 \end{cases}$$