Method of undetermined coeff's for $y_p(t)$ 1/28/12

— refined version:

$$ay'' + by' + cy = f(t)$$

Step 1: Solve char. eq^{n}

$$ar^2 + br + c = 0 \implies r_1, r_2 \text{ two roots}$$

(we need this for y_h anyway.)

Step 2: Examine $f(t)$ to determine the form of $y_p(t)$:

— If $f(t) = (a_n t^n + \cdots + a_1 t + a_0)\, e^{kt}$

($k=0$ if no exp. term)

then

fudge factor

$$y_p(t) = \begin{cases} (A_n t^n + \cdots + A_1 t + A_0)\, e^{kt} & \text{if } k \neq r_1,\ \neq r_2 \\ t\,(A_n t^n + \cdots + A_1 t + A_0)\, e^{kt} & \text{if } k = \text{one but not both of } r_1, r_2 \\ t^2\,(A_n t^n + \cdots + A_1 t + A_0)\, e^{kt} & \text{if } k = r_1 = r_2 \end{cases}$$

increase the degree of polynomial by 1.

increase the degree by 2.

— If $f(t) = (a_n t^n + \cdots + a_1 t + a_0)\, \underline{e^{kt} \cos \omega t}$
$+ (b_n t^n + \cdots + b_1 t + b_0)\, e^{kt} \sin \omega t \quad (\omega \neq 0)$

then

$$y_p(t) = \begin{cases} (A_n t^n + \cdots + A_1 t + A_0)\, e^{kt} \cos \omega t + (B_n t^n + \cdots + B_1 t + B_0)\, e^{kt} \sin \omega t & \text{if } \underline{\underline{k \pm i\omega}} \neq r_1, r_2 \\ t\left[(A_n t^n + \cdots + A_1 t + A_0)\, e^{kt} \cos \omega t + (B_n t^n + \cdots + B_1 t + B_0)\, e^{kt} \sin \omega t\right] & \text{if } k \pm i\omega = \text{one of } r_1, r_2 \end{cases}$$

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— If $f(t) =$ sum, then $y_p(t) =$ sum.

e.g. If $f(t) = f_1(t) + f_2(t)$, ~~then~~ f_1, f_2 diff type

then $y_p(t) = y_{1,p}(t) + y_{2,p}(t)$ w7.

$y_{1,p}$ particular sol$^{\underline{n}}$. for

$$ay'' + by' + cy = \underline{\underline{f_1(t)}}$$

$y_{2,p}$ particular sol$^{\underline{n}}$. for

$$ay'' + by' + cy = \underline{\underline{f_2(t)}}$$

Final complex note:

$r = \alpha \pm i\beta \rightsquigarrow e^{\alpha t}(C_1 \cos \beta t + C_2 \sin \beta t)$

$k \pm i\omega \longleftarrow e^{kt}(A \cos \underline{\underline{\omega}} t + B \sin \underline{\underline{\omega}} t)$

Trig identities:

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$$\cos u - \cos v = -2 \sin \frac{u-v}{2} \sin \frac{u+v}{2}$$

since

$$\cos(a+b) = \cos a \cos b - \sin a \sin b$$

$$\cos(a-b) = \cos a \cos b + \sin a \sin b$$

Connection with Euler's formula:

$$e^{i\theta} = \cos\theta + i \sin\theta$$

$$\Rightarrow \quad \cos(a+b) + i \sin(a+b)$$

$$= e^{i(a+b)} = e^{ia} \cdot e^{ib}$$

$$= (\cos a + i \sin a)(\cos b + i \sin b)$$

$$= [\cos a \cos b - \sin a \sin b]$$

$$+ i [\sin a \cos b + \cos a \sin b]$$