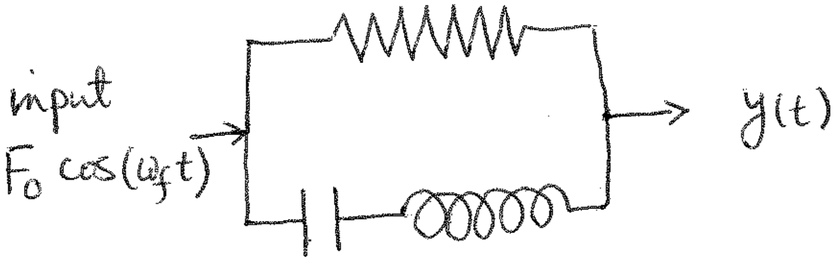
01/25/12

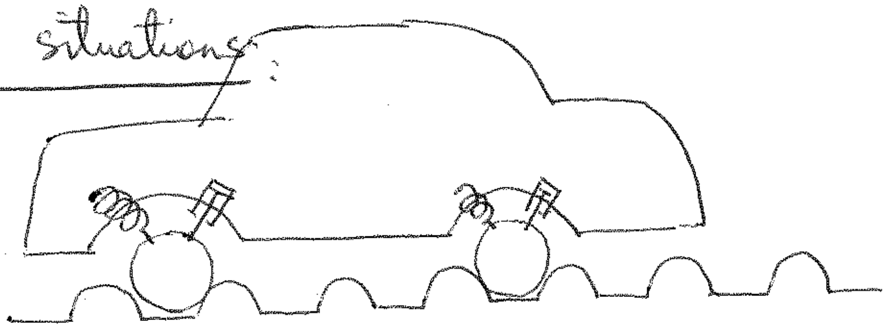
<u>Last time</u>: Forced Oscillations

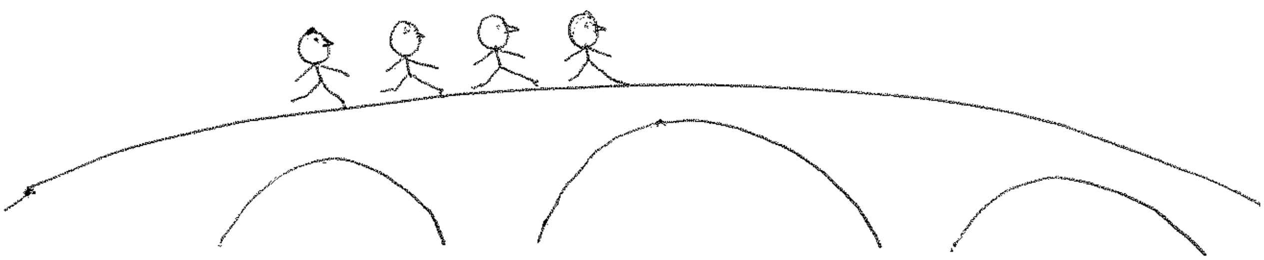
forcing amplitude, forcing frequency

$$(\star) \qquad ay'' + by' + cy = \underbrace{F_0 \cos(\omega_f t)}_{\text{periodic forcing term}}$$



<u>Similar situations</u>:



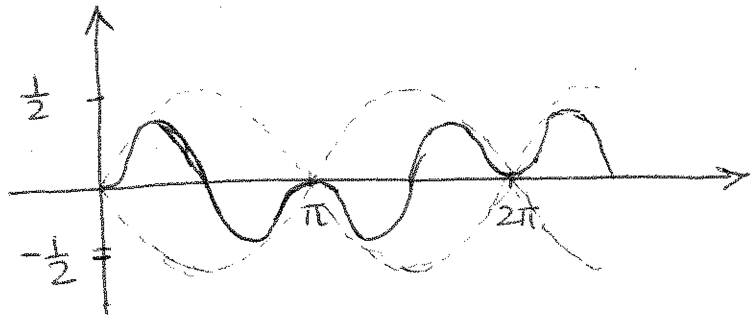


To understand these phenomena $\rightsquigarrow$ solving $(\star)$

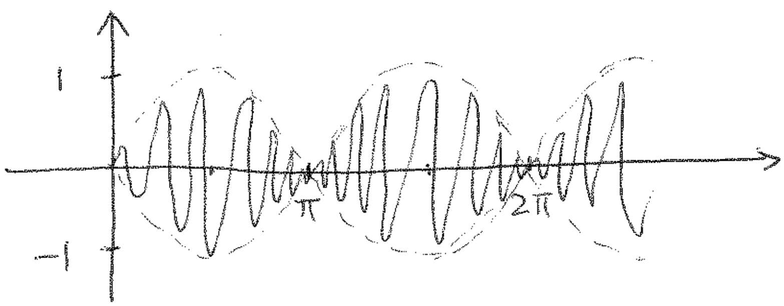
<u>Ex</u>: $\ddot{x} + x = 2\cos 3t, \quad x(0)=0, \quad x'(0)=0$

$x(t) = \frac{1}{2} \sin t \sin 2t \left(= \frac{1}{4} (\cos t - \cos 3t) \right)$

01/25/12



$x(t) = \sin t \sin 20t \left(= \frac{1}{2} (\cos(19t) - \cos(21t)) \right)$



beats.

1) Undamped $\quad \underline{m\ddot{x} + kx = F_0 \cos \omega_f t}$

natural frequency $\omega_0 = \sqrt{\frac{k}{m}}$. forcing freq. ω_f.

$\omega_0 \neq \omega_f$ $\quad$ sol$^{\underline{n}}$ $x(t)$ = sum of two sinusoidal f$^{\underline{n}}$s of different frequencies $\rightsquigarrow$ beats

$\omega_0 = \omega_f$ $\quad$ sol$^{\underline{n}}$ $x(t)$ = sinusoidal f$^{\underline{n}}$ of growing amplitude $\rightsquigarrow$ pure resonance

2) damped $\quad \underline{m\ddot{x} + b\dot{x} + kx = F_0 \cos \omega_f t}$

sol$^{\underline{n}}$ $x(t) = \underbrace{x_n(t)}_{\text{transcient sol}^{\underline{n}}} + \underbrace{x_p(t)}_{\text{steady-state sol}^{\underline{n}}}$