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For the pass 3 weeks:

$$\underbrace{a y'' + b y' + c y} = f(t)$$

linear $\Longrightarrow$ $y = y_h + y_p$

Power of abstraction:

$$\underset{\text{input}}{y} \longrightarrow \boxed{\ T\ } \longrightarrow \underset{\text{output}}{T(y)} = a y'' + b y' + c y$$

Linearity means

$$k y \longrightarrow \boxed{\ T\ } \longrightarrow k\,T(y) \qquad \text{i.e. } T(ky) = k\,T(y)$$

$$y_1 + y_2 \longrightarrow \boxed{\ T\ } \longrightarrow T(y_1) + T(y_2) \qquad \text{i.e. } T(y_1 + y_2) = T(y_1) + T(y_2)$$

$\rightsquigarrow$ linear transformation

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$$Ty = ay'' + by' + cy$$

$$y \longrightarrow \boxed{\;T\;} \longrightarrow Ty$$

$$ky \longrightarrow \boxed{\;T\;} \longrightarrow T(ky) = a(ky)'' + b(ky)' + c(ky)$$

$$= aky'' + bky' + cky$$

$$= k(ay'' + by' + cy)$$

$$= kT(y)$$

$$y_1 \longrightarrow \boxed{\;T\;} \longrightarrow Ty_1 = ay_1'' + by_1' + cy_1$$

$$y_2 \longrightarrow \boxed{\;T\;} \longrightarrow Ty_2 = ay_2'' + by_2' + cy_2$$

$$y_1 + y_2 \longrightarrow \boxed{\;T\;} \longrightarrow T(y_1 + y_2) = a(y_1 + y_2)'' + b(y_1 + y_2)' + c(y_1 + y_2)$$

$$= a(y_1'' + y_2'') + b(y_1' + y_2') + c(y_1 + y_2)$$

$$= (ay_1'' + by_1' + cy_1) + (ay_2'' + by_2' + cy_2)$$

$$= Ty_1 + Ty_2.$$

V, W vector spaces 01/27/12

$T: V \longrightarrow W$ is a linear transformation if it preserves both the vector addition and scalar multiplication, i.e. $\forall\, u, v \in V$, $\forall\, k$ scalar

$$\boxed{\begin{aligned} T(u+v) &= T(u) + T(v) \\ T(ku) &= k\,T(u) \end{aligned}}$$

$$\Longleftrightarrow \quad T(k_1 u_1 + k_2 u_2) = k_1 T(u_1) + k_2 T(u_2)$$

$$\Longleftrightarrow \quad \boxed{T(k_1 u_1 + \cdots + k_n u_n) = k_1 T(u_1) + \cdots + k_n T(u_n)}$$

Linear transf$^{\underline{n}}$'s and matrices:

$T: \mathbb{R}^n \longrightarrow \mathbb{R}^m$ linear transf$^{\underline{n}}$ $\longleftrightarrow$ $m \times n$ matrix $A = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix}$

$$T\begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = A \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$$

Standard matrix of T:

$$A = \left[\, T(e_1) \mid T(e_2) \mid \cdots \mid T(e_n) \,\right]$$

$e_1 = (1, 0, \cdots, 0)$, $e_2 = (0, 1, \cdots, 0)$, $e_n = (0, 0, \cdots, 1)$ standard basis of $\mathbb{R}^n$.