

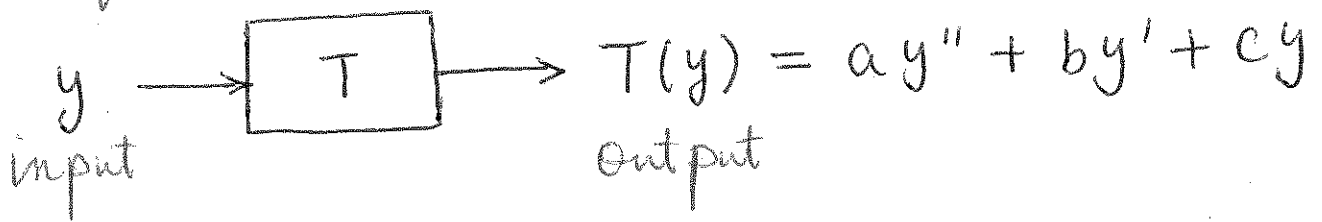
For the pass 3 weeks:

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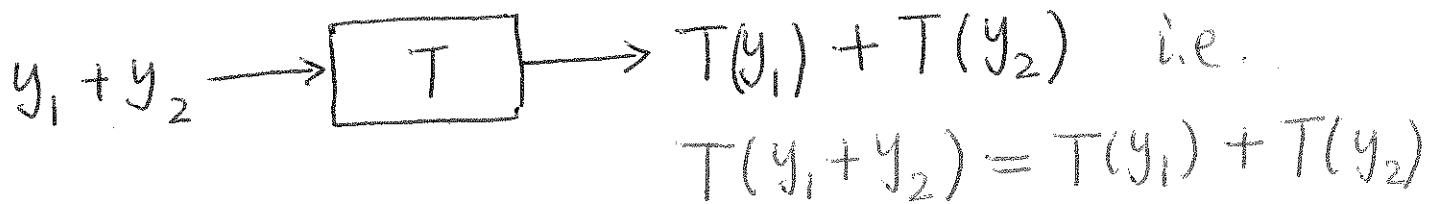
$$ay'' + by' + cy = f(t)$$

linear $\Rightarrow y = y_h + y_p$

Power of abstraction:



Linearity means



\rightsquigarrow linear transformation

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$$Ty = ay'' + by' + cy$$



$ky \rightarrow \boxed{T} \rightarrow T(ky) = a(ky)'' + b(ky)' + c(ky)$
 $= ak y'' + bk y' + ck y$
 $= k(ay'' + by' + cy)$
 $= kT(y)$

$y_1 \rightarrow \boxed{T} \rightarrow Ty_1 = ay_1'' + by_1' + cy_1$

$y_2 \rightarrow \boxed{T} \rightarrow Ty_2 = ay_2'' + by_2' + cy_2$

$y_1 + y_2 \rightarrow \boxed{T} \rightarrow T(y_1 + y_2) = a(y_1 + y_2)'' + b(y_1 + y_2)' + c(y_1 + y_2)$
 $= a(y_1'' + y_2'') + b(y_1' + y_2') + c(y_1 + y_2)$
 $= (ay_1'' + by_1' + cy_1) + (ay_2'' + by_2' + cy_2)$
 $= Ty_1 + Ty_2$

V, W vector spaces

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$T: V \rightarrow W$ is a linear transformation

if it preserves both the vector addition and scalar multiplication, i.e. $\forall u, v \in V, \forall k$ scalar

$$\begin{aligned} T(u+v) &= T(u) + T(v) \\ T(ku) &= kT(u) \end{aligned}$$

$$\iff T(k_1 u_1 + k_2 u_2) = k_1 T(u_1) + k_2 T(u_2)$$

$$\iff T(k_1 u_1 + \dots + k_n u_n) = k_1 T(u_1) + \dots + k_n T(u_n)$$

Linear transfⁿs and matrices:

$$T: \mathbb{R}^n \rightarrow \mathbb{R}^m$$

linear transfⁿ

$$\iff m \times n \text{ matrix } A = \begin{bmatrix} a_{11} & \dots & a_{1n} \\ \vdots & & \vdots \\ a_{m1} & \dots & a_{mn} \end{bmatrix}$$

$$T \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = A \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$$

Standard matrix of T :

$$A = [T(e_1) \mid T(e_2) \mid \dots \mid T(e_n)]$$

$$e_1 = (1, 0, \dots, 0), e_2 = (0, 1, \dots, 0), e_n = (0, 0, \dots, 1)$$

standard basis of \mathbb{R}^n