02/06/12

<u>System of DE</u>: $\vec{x}' = A\vec{x}$

$$\vec{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}, \quad A = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{n1} & \cdots & a_{nn} \end{bmatrix}$$

In general, coupled!

When $A = \begin{bmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & \lambda_n \end{bmatrix}$ <u>diagonal matrix</u>

The system becomes $\begin{cases} x_1' = \lambda_1 x_1 \\ \vdots \\ x_n' = \lambda_n x_n \end{cases}$ uncoupled!

<u>Diagonalization</u>: How to make a matrix into a diagonal matrix?

Let $\lambda_1, \lambda_2, \cdots, \lambda_n$ be <u>eigenvalues</u> of A w/ corresponding <u>eigenvectors</u> $\vec{v}_1, \vec{v}_2, \cdots, \vec{v}_n$.

Assume that $\vec{v}_1, \vec{v}_2, \cdots, \vec{v}_n$ are linearly indep. (This happens if all eigenvalues are distinct)

$\Rightarrow P = [\vec{v}_1 \mid \vec{v}_2 \mid \cdots \mid \vec{v}_n]$ invertible and

$$P^{-1}AP = \begin{bmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \ddots & \\ & & & \lambda_n \end{bmatrix} \quad \left(\Leftrightarrow AP = P \begin{bmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \ddots & \\ & & & \lambda_n \end{bmatrix} \right.$$

$$\Leftrightarrow \left. \begin{cases} A\vec{v}_1 = \lambda_1 \vec{v}_1 \\ \vdots \\ A\vec{v}_n = \lambda_n \vec{v}_n \end{cases} \right)$$