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Diagonization: A $n \times n$ matrix

$\lambda_1, \lambda_2, \cdots, \lambda_n$ eigenvalues

$\vec{v}_1, \vec{v}_2, \cdots, \vec{v}_n$ eigenvectors linearly indep

$\Rightarrow$ $P = [\vec{v}_1 | \vec{v}_2 | \cdots | \vec{v}_n]$ diagonizes A:

$$\boxed{P^{-1}AP = \begin{bmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_n \end{bmatrix}}$$

Q: When can we find a full set of linearly indep. eigenvectors?

Distinct Eigenvalues Theorem: If all eigenvalues are distinct, the corresponding eigenvectors are linearly indep. (and hence gives a full set of linearly indep. eigenvectors).

Rk: But even with multiple eigenvalues (identical eigenvalues) you may still get a full set of linearly indep. eigenvectors

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Coordinate transf$^{\underline{n}}$:

Systems of DE: $\vec{x}' = A\vec{x}$

can be decoupled by coordinate transf$^{\underline{n}}$

$$\vec{x} = P\vec{u}$$

$$\Rightarrow \quad \vec{u}' = \begin{bmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_n \end{bmatrix} \vec{u}$$

Ch 6: Linear System of DE's

$$\boxed{\vec{x}' = A(t)\vec{x} + \vec{f}(t)}$$

$\vec{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$, $\vec{f}(t) = \begin{bmatrix} f_1(t) \\ \vdots \\ f_n(t) \end{bmatrix}$

$A(t) = \begin{bmatrix} a_{11}(t) & \cdots & a_{1n}(t) \\ \vdots & & \vdots \\ a_{n1}(t) & \cdots & a_{nn}(t) \end{bmatrix}$

$\Rightarrow$ general sol$^{\underline{n}}$

$$\vec{x} = \vec{x}_h(t) + \vec{x}_p(t)$$

$\vec{x}_p(t)$: a particular sol$^{\underline{n}}$.

$\vec{x}_h(t)$: homog. sol$^{\underline{n}}$, i.e., solves

$$\vec{x}' = A(t)\vec{x}.$$

Structure Thm: Assume that $A(t)$ is continuously diff. $\Rightarrow$ the set of sol$^{\underline{n}}$s of

$$\vec{x}' = A(t)\vec{x}$$

is a vector space of dim $= n$. I.e., there are n linearly indep. sol$^{\underline{n}}$s $\vec{x}_1(t), \cdots, \vec{x}_n(t)$ s.t. the general sol$^{\underline{n}}$ is

$$\vec{x}(t) = c_1\vec{x}_1(t) + \cdots + c_n\vec{x}_n(t)$$