02/13/12

Last time:

System of DE's:

$$\vec{x}' = A\vec{x} \qquad \vec{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}, \quad A = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{n1} & \cdots & a_{nn} \end{bmatrix}$$

general sol$^{\underline{n}}$

$$\vec{x}(t) = c_1 \vec{x}_1(t) + \cdots + c_n \vec{x}_n(t)$$

for $\underline{n}$ linearly indep. sol$^{\underline{n}}$'s $\vec{x}_1(t), \cdots, \vec{x}_n(t)$.

How to find $\vec{x}_1(t), \cdots, \vec{x}_n(t)$?

$\boxed{\vec{x}(t) = e^{\lambda t}\vec{v} \text{ is a sol}^{\underline{n}}}$ $\iff$ $\boxed{A\vec{v} = \lambda\vec{v} \text{ i.e. } \lambda \text{ is an eigenvalue and } \vec{v} \text{ a corresponding eigenvector}}$

$\Longrightarrow$

If $\lambda_1, \cdots, \lambda_n$ eigenvalues of A

$\vec{v}_1, \cdots, \vec{v}_n$ linearly indep. eigenvectors

then the general sol$^{\underline{n}}$

$$\vec{x}(t) = c_1 e^{\lambda_1 t}\vec{v}_1 + \cdots + c_n e^{\lambda_n t}\vec{v}_n.$$

Short of (linearly indep) eigenvectors?

Repeated eigenvalues : $\lambda_1 = \lambda_2$ but only one eigenvector $\vec{v}_1$

$\Rightarrow$ a linearly indep new sol$^{\underline{n}}$ is given by

$$\boxed{t e^{\lambda_1 t} \vec{v}_1 + e^{\lambda_1 t} \vec{u}}$$

where $\vec{u}$ is a sol$^{\underline{n}}$ of

$$\boxed{(A - \lambda_1 I)\vec{u} = \vec{v}_1}$$