Last time

Stability of System of Linear Homog. DE: (02/24/12)

$$\boxed{\vec{x}' = A\vec{x}}$$

- A $\text{sol}^{\underline{n}}$. $\vec{x}(t)$ represents <u>a curve</u> in $\mathbb{R}^n$ whose tangent vectors are given by $A\vec{x}$ at $\vec{x}$.

- The zero $\text{sol}^{\underline{n}}$. $\vec{x}(t) \equiv 0$ represents ~~Ø~~ a constant curve —— <u>equilibrium $\text{sol}^{\underline{n}}$</u>.

- An equilibrium $\text{sol}^{\underline{n}}$ is called <u>stable</u> if all nearby $\text{sol}^{\underline{n}}\text{s}$ (i.e. nearby curves) stay bounded.

- Stability here completely determined by eigenvalues (and eigenvectors) of A:

  <u>stable</u> if all eigenvalues are negative or have negative real parts;

  <u>unstable</u> if one eigenvalue ~~ha~~ is positive or has positive real part.

<u>Borderline Case</u>: next time.

Stability — borderline case (02/24/12)

$$\vec{x}' = A\vec{x} \qquad A \quad 2\times 2.$$

1). $\lambda_1 = 0$ but $\lambda_2 < 0 \Rightarrow$ stable

2). $\lambda_1 = \lambda_2 = 0$ but only one eigenvector (always the case unless $A = 0$) $\Rightarrow$ unstable

3). $\lambda_1 = i\beta$, $\beta \neq 0$. $(\Rightarrow \lambda_2 = -i\beta) \Rightarrow$ stable

Phase Portrait of Linear Homog System Revisited

$$\vec{x}' = A\vec{x}$$

1). Along eigendirections, the vector will be pointing in the same direction if eigenvalue > 0; in the opposite direction if eigenvalue < 0.

2). For a general direction $\vec{v}$, the vectors at different point along the direction $\vec{v}$ all points to the same direction $A\vec{v}$; At points in the reverse direction $-\vec{v}$, the vectors are all pointing in the opposite direction $-A\vec{v}$.