

Last time: Nonlinear System of DE. 02/29/12

Autonomous System:  $\begin{cases} x' = f(x, y) \\ y' = g(x, y) \end{cases}$  No explicit  $f^{\text{th}}$  of  $t$ !

① Equilibrium:

$$\begin{cases} f(x, y) = 0 \\ g(x, y) = 0 \end{cases}$$

Corresponds to constant curve  
 $x = x_0, y = y_0$

② Nullclines:

v-nullcline:  $\underline{f(x, y) = 0}$

Vectors are vertical  
along v-nullcline.

h-nullcline:  $\underline{g(x, y) = 0}$

Vectors are horizontal  
along h-nullcline.

③ v-nullcline and h-nullcline intersect at the equilibria.

④ v-nullcline + h-nullcline divide the phase plane into 4 types of regions

a).  $x' < 0, y' < 0$   pointing SW

b).  $x' < 0, y' > 0$   pointing NW

c).  $x' > 0, y' > 0$   pointing NE

d).  $x' > 0, y' < 0$   pointing SE.

## Additional tips

sol<sup>n</sup> curves or trajectories

02/29/12

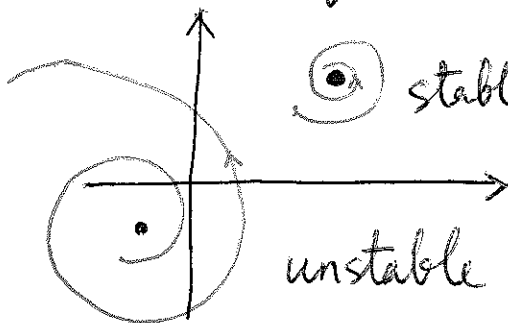
- Curves will not cross ("unless at the equilibria") provided uniqueness holds.
- Assuming  $f(x,y)$ ,  $g(x,y)$  are continuous, curves will be continuous (will not break) and smooth (will not bend or kink) ("except at the equilibria")
- Curves will usually "start" somewhere (equilibrium,  $\infty$  etc) and "end" somewhere (equilibrium,  $\infty$ , etc.)
- New for Nonlinear system: limit cycles — a closed curve (representing a periodic sol<sup>n</sup>) to which other sol<sup>n</sup>s either wind into or away from it.

## Stability

stable

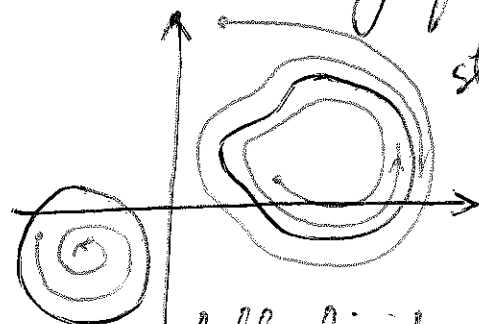
unstable

An equilibrium or limit cycle is  
if all nearby curves stay close to it;  
if some! nearby curves wander away from it.



stable equilibri

unstable equilibri



stable limit cycle

unstable limit cycle