Last time: Nonlinear System of DE. 02/29/12

Autonomous System: $\begin{cases} x' = f(x,y) \\ y' = g(x,y) \end{cases}$ No explicit t!

(1) Equilibrium:

$\begin{cases} f(x,y) = 0 \\ g(x,y) = 0 \end{cases}$

Corresponds to constant curve $x = x_0$, $y = y_0$

(2) Nullclines:

v-nullcline: $f(x,y) = 0$. Vectors are vertical along v-nullcline

h-nullcline: $g(x,y) = 0$. Vectors are horizontal along h-nullcline.

(3) v-nullcline and h-nullcline intersect at the equilibria.

(4) v-nullcline + h-nullcline divide the phase plane into 4 types of regions

a) $x' < 0$, $y' < 0$  pointing SW

b) $x' < 0$, $y' > 0$  pointing NW

c) $x' > 0$, $y' > 0$  pointing NE

d) $x' > 0$, $y' < 0$  pointing SE

Additional tips

Sol^{n} curves or trajectories

02/29/12

- Curves will not cross ("unless at the equilibria") provided uniqueness holds.
- Assuming $f(x,y)$, $g(x,y)$ are continuous, curves will be continuous (will not break) and smooth (will not bend or kink) ("except at the equilibria")

- Curves will usually "start" somewhere (equilibrium, ∞ etc) and "end" somewhere (equilibrium, ∞, etc.)
- New for Nonlinear system: <u>limit cycles</u> — a closed curve (representing a periodic sol^{n}) to which other sol^{n}s either wind into or away from it.

Stability An equilibrium or limit cycle is

stable if all nearby curves stay close to it;

unstable if some nearby curves wander away from it.